ACI MATERIALS JOURNAL TECHNICAL PAPER

Title no. 103-M03

# Evaluation of Ratio between Splitting Tensile Strength and Compressive Strength for Concretes up to 120 MPa and its Application in Strength Criterion

by Nihal Arıoglu, Z. Canan Girgin, and Ergin Arıoglu

*A large-scale regression analysis was carried out using experimental data gathered from various sources to evaluate the ratio of splitting tensile strength to cylinder compressive strength as a function of compressive strength of concrete. The reliability of the proposed equation based on experimental data for compressive strength ranging between 4 and 120 MPa (580 to 17,400 psi) was assessed by means of the integral absolute error (IAE). It is also shown that, by only knowing compressive strength and the ratio of tensile to compressive strength, the failure envelope for very high-strength concrete can be established using Johnston's strength criterion without performing triaxial compression tests. A numerical example demonstrates the application of Johnston's strength criterion.*

**Keywords:** compressive strength; confinement; high-strength concrete; splitting tensile strength; stress.

## INTRODUCTION

The ratio between tensile strength and compressive strength is an important material property of concrete. The value of this ratio is required for the following applications:

1. With respect to Bortolotti's studies,[1,2] the ultimate strain value in uniaxial tension is expressed in terms of this strength ratio.

2. According to Johnston's strength criterion[3] for intact rock under triaxial compression, the material constants defining the failure envelope are related to the ratio of compressive strength to tensile strength. Reported results of Setunge et al.[4] and Yapı Merkezi[5] for very high-strength concrete in triaxial compression are in good agreement with the strength criterion proposed by Johnston.

3. There are three types of tests to measure strength in tension: direct tension, flexure, and splitting tension.[6] It has been well established that the simplest and the most reliable method, which generally provides a lower coefficient of variation, is the splitting tensile test[7-9] of a cylindrical specimen. In this test, a cylindrical specimen is loaded in compression diametrically between two plates. According to the theory of elasticity, this loading generates almost uniform tensile stress along the diameter, which causes the specimen to fail by splitting along a vertical plane. The splitting strength $f_{tsp}$ can be used to estimate direct tensile strength $f_t$ by multiplying by a conversion factor of $\lambda = 0.9$, as given in the CEB-FIB Code[10] and by Hannant et al.[7]

The objectives of the investigation reported herein are as follows:

1. To evaluate the ratio of splitting tensile strength to compressive strength ($f_{tsp}/f_c$) as a function of cylinder compressive strength of concrete $f_c$ by means of regression analysis of experimental data from the literature.[11-13] The data gathered embrace a variety of cements (normal portland cement, rapid-hardening portland cement), several supplementary cementitious materials (fly ash, bottom ash, silica fume), various water-cementitious material ratios (*w/cm*) ranging from 0.24 to 0.55, testing ages from 1 to 360 days, curing temperatures from 0 to 30 °C (32 to 86 °F), and several moisture conditions. Concrete compressive strength varies from 4 MPa to approximately 120 MPa (580 to 17,400 psi). To confirm the derived relationship, test data from other sources[8,14-17] were used for verification.

2. To verify whether Johnston's strength criterion is valid for high-strength concretes by making use of the derived relationship between the ratio of splitting tensile strength to compressive strength ($f_{tsp}/f_c$) and the cylinder compressive strength $f_c$ in this study.

## RESEARCH SIGNIFICANCE

This study introduces a relationship between the ratio of splitting tensile strength to compressive strength ($f_{tsp}/f_c$) and the cylinder compressive strength $f_c$, which is applicable to concrete at early ages (12 hours and longer) as well as very high-strength concrete (up to 120 MPa [17,400 psi]). Existing relationships in the literature are based mainly on data obtained from concretes with compressive strength of not more than 83 MPa (12,000 psi). The reliability of the proposed equation is assessed on the basis of integral absolute error (IAE, %). The results of this analysis are particularly important because no comprehensive information on the reliabilities of the relationships used in the current building codes has been available.

In the design of triaxially compressed structures, it is necessary to have an expression relating the ultimate strength $f_1$ and the confining pressure $f_r$. Johnston[3] proposed an empirical strength criterion based on the ratio of compressive to tensile strength and the confinement effectiveness, for a range of geomaterials. This study shows that Johnston's strength criterion[3] can be used to adequately predict the ultimate strength of very high-strength concrete under triaxial compression. Knowledge of the ratio between splitting tensile strength and uniaxial compressive strength should allow for the estimation of strength of very high-strength concrete under confinement. Furthermore, this knowledge could reduce costs associated with triaxial testing programs for very high-strength concrete.

*ACI Materials Journal*, V. 103, No. 1, January-February 2006.
MS No. 04-097 received March 30, 2004, and reviewed under Institute publication policies.  Pertinent discussion including authors' closure, if any, will be published in the November-December 2006 *ACI Materials Journal* if the discussion is received by August 1, 2006.

**Nihal Arıoglu** *is an associate professor in the Architecture Faculty, Istanbul Technical University, Istanbul, Turkey. She received her PhD from the Yildiz Technical University in 1993. Her research interests include evaluation of building elements, materials, and mechanical properties of rocks used for restoration projects.*

**Z. Canan Girgin** *is a research engineer in the R&D Department, Yapı Merkezi Inc., Istanbul, Turkey. She received her PhD from the Istanbul Technical University in 1996. Her research interests include structural engineering, soil-structure interaction, masonry structures, mechanical properties of concrete, self-consolidating concrete, and fiber-reinforced concrete.*

**Ergin Arıoglu** *is a professor emeritus of mining engineering at Istanbul Technical University. He is the Director of the R&D Department, Yapı Merkezi Inc. He received his PhD from the University of Newcastle Upon Tyne, England, in 1976. His research interests include design of artificial reinforced roof in thick coal seam workings, mechanical properties of engineered back-filling materials, and very high-strength concrete.*

## RESULTS OF REGRESSION ANALYSIS AND DISCUSSION

### General

In recent studies,[4,5] the failure envelope for very high-strength concrete subjected to confining pressure was shown to be in reasonable agreement with Johnston's strength criterion.[3] If accurate estimates for very high-strength concrete under confinement are required, the relationship between the strength ratio and the compressive strength must be established by means of the regression analysis. The following factors must be taken into account in this analysis:

1. The mathematical model should be based on physically significant parameters. Furthermore, it should be as simple as possible and easily usable in any analysis.

2. The relationship should be applicable over a wide range of experimental data.

3. The coefficient of correlation $r$ that measures the strength of the proposed relationship should be large. It is important to note that even when the correlation is significant, the variability can still be large, and the proposed equation may not be reliable.[18]

4. The accuracy of the relationship should be as high as possible. In other words, the errors associated with the regression model should be as small as possible.

**Table 1—Brief descriptions of main data—221 test data points—used for regression analysis**

| Mixture properties | Gardner[11] | Gardner et al.[12] | Imam et al.[13] |
|---|---|---|---|
| Mixture properties | | | |
| Mixture numbers | 6 | 6 | 18 |
| Cement type | Type I, II | Type III | P 50* |
| Cement quantity, kg/m$^3$ | 225 to 409 (Type I)<br>307, 414 (Type III) | 304 to 411 | 410 to 550 |
| Fly ash, kg/m$^3$ | 75,101 (two mixtures)<br>— (four mixtures) | — | — |
| Silica fume, kg/m$^3$ | — | — | 0 to 82.5 |
| Total cementitious materials, kg/m$^3$ | 300 to 414 | 304 to 411 | 410 to 632.5 |
| Coarse aggregate, kg/m$^3$ | 1040 to 1080† | 1039 to 1058,† crushed limestone<br>($D_{max}$ = 25 mm) | 0 to 1097 (gravel)<br>0 to 1375 (porphyry) |
| Sand, kg/m$^3$ | 780 to 878 | 780 to 832,† natural sand | 430 to 720 |
| Water, kg/m$^3$ | 138 to 169 | 143 to 169 | 119 to 152 |
| High-range water-reducing admixture, kg/m$^3$ | 0 to 2.1 (N) | 2.85 (for $w/cm$ = 0.35) (N), N/A (for others) | 12.3 to 22.1 |
| Air-entraining agent, mL/m$^3$ | — | 360 to 493 | — |
| Physical properties after mixing and molding | | | |
| Density, kg/m$^3$ | — | 2343 to 2435 | 2370 to 2465 |
| Slump, mm | 50 to 100 | 38 to 64 | 10 to 240 |
| Air content, % | N/A (five mixtures)<br>3 (one mixture) | 4.0 to 5.8 | — |
| Ratios (by mass) | | | |
| Fly ash/(cement + fly ash) | 0.25 (two mixtures)<br>0 (four mixtures) | — | — |
| Silica fume/(cement + silica fume) | — | — | 0 to 13% |
| $w/cm$ | 0.55, 0.35 (each one is three mixtures) | 0.55, 0.45, 0.35 (each one is two mixtures) | 0.24 to 0.29 |
| Total aggregate/cementitious materials | 4.5 to 6.2 | 4.5 to 6.1 | 3.0 to 3.8 |
| Curing conditions | | | |
| Curing temperatures | Each mixture cured in water tanks at 0, 10, 20, and 30 °C | Three mixtures cured in seawater at 0 °C, three mixtures cured in moist chamber at 22 °C | Fog room 20 ± 2 °C, 95 ± 2% RH |
| Testing time | 1 to 112 days | 3 to 360 days | 28 days |
| Ranges of cylinder compressive and splitting tensile strengths | | | |
| Compressive strength, MPa (psi) | 4.0 to 56.7‡ (573 to 8223) | 13.4 to 69.9 (1943 to 10,138) | 82 to 117.4 (11,893 to 17,027) |
| Splitting tensile strength, MPa (psi) | 0.77 to 4.92‡ (111 to 713) | 2.1 to 5.83 (304 to 845) | 5.94 to 7.74 (861 to 1122) |

*Portland cement with compressive strength ≥ 50 MPa at 28 days according to EN 196.
†Saturated surface dry.
‡Compressive strength and splitting tensile strength values are average of five specimens.
Note: N = naphtalene-based; 1 kg = 2.205 lb; 1 kg/m$^3$ = 1.685 lb/yd$^3$; 1 L = 0.220 gal.; and 1 MPa = 145.038 psi.

In this study, the reliability of the relationships derived from the regression analysis was assessed on the basis of the integral absolute error (IAE, %). This index has been used by others[11,19,20] to evaluate the goodness of fit of proposed relationships, and it is computed from Eq. (1)

$$\mathrm{IAE} = \sum \frac{[(O_i - P_i)^2]^{1/2}}{\Sigma O_i} \cdot 100 \qquad (1)$$

where $O_i$ is the observed value, and $P_i$ is the predicted value from the regression equation. The IAE measures the relative deviations of data from the regression equation. When the IAE is zero, the predicted values from the regression equation are equal to the observed values; this situation rarely occurs. When comparing different equations, the regression equation having the smallest value of the IAE can be judged as the most reliable. A range of the IAE from 0 to 10% may be regarded as the limits for an acceptable regression equation.

### Brief presentation of experimental data

The main sources of data used for the regression analysis along with the characteristics of the concretes (type of cement, cement quantity, *w/cm*, type of supplementary cementitious material, curing temperatures, and testing age) are given in Table 1. As seen from Table 1, the data[11-13] collected from the literature is representative of the diversity that may occur in concrete construction. The cylinder compressive strength $f_c$ varies from approximately 4 to 120 MPa (580 to 17,400 psi). In other words, the regression analysis was based on data ranging from immature concrete to very high-strength concrete.

### Results of regression analyses

To evaluate the ratio of splitting tensile strength to compressive strength $f_{tsp}/f_c$, a series of regression analyses was undertaken and the results of these analyses are summarized in Table 2. The values of the IAE computed for the regression equations given in Table 2 are compiled in Table 3. From Table 2 and 3, the following observations can be made:

1. Based on the coefficient of correlation $r$, Eq. (4) to (7) provide equally strong relationships between the ratio of splitting tensile to compressive strength ($f_{tsp}/f_c$) and compressive strength $f_c$. In terms of relative error, Eq. (6) and (7) provide the smallest values of IAE. Thus these are several equations with similar reliability. The power function, Eq. (7), can be selected for its simplicity without loss of accuracy.

2. Using the control data, the value of IAE for Eq. (7) was found to be 7.14%. Equation (7) is also in a close agreement with the control test data ranging between 6 to 122 MPa (870 to 17,690 psi). In brief, Eq. (7) shows a better accuracy for predicting the splitting tensile strength.

### Table 2—Results of several statistical models used in regression analyses

| Equation | Statistical model | *A* | *B* | *r* |
|---|---|---|---|---|
| (2) | $\frac{f_{tsp}}{f_c} = \frac{Af_c}{B + f_c}$ | 0.08559 | –4.3450 | 0.838 |
| (3) | $\log\frac{f_{tsp}}{f_c} = Af_c + B$ | –0.003707 | –0.8338 | 0.888 |
| (4) | $\frac{f_{tsp}}{f_c} = \frac{1}{A + Bf_c}$ | 6.2739 | 0.08814 | 0.967 |
| (5) | $\frac{f_{tsp}}{f_c} = A\frac{1}{\sqrt{f_c}} + B$ | 0.3898 | 0.03756 | 0.968 |
| (6) | $\frac{f_{tsp}}{f_c} = A\log f_c + B$ | –0.09337 | 0.2509 | 0.971 |
| (7) | $\frac{f_{tsp}}{f_c} = Af_c^{\,B}$ | 0.3870 | –0.3700 | 0.951 |

Note: *A*,*B* = constants of regression equation; *r* = correlation coefficient of regression equation; and *n* = 221, corresponding to group (I).

### Table 3—Calculated values of IAE for equations given in Table 2

| Equation | IAE, % (I) | IAE, % (II) | IAE, % (III) |
|---|---|---|---|
| (2) | 22.81 | 12.45 | 19.60 |
| (3) | 7.34 | 8.58 | 7.72 |
| (4) | 6.37 | 7.49 | 6.72 |
| (5) | 5.97 | 7.48 | 6.44 |
| (6) | 5.55 | 7.67 | 6.21 |
| (7) | 5.60 | 7.14 | 6.07 |

Note: IAE calculated for: (I) one group data used for regression analysis (*n* = 221) (range: 4 to 118 MPa [580 to 11,115 psi]);[11-13] (II) other group data used as "control data" to assess accuracy of derived regression equations (*n* = 104) (range: 6 to 122 MPa [870 to 17,690 psi]).[8,14-17] This group of data was selected randomly; (III) = (I) + (II) corresponding to all data (*n* = 325) (range: 4 to 122 MPa [580 to 17,690 psi]),[8,11-17] where IAE equals integral absolute error, *n* equals number of data test points, and 1 MPa = 145.038 psi.

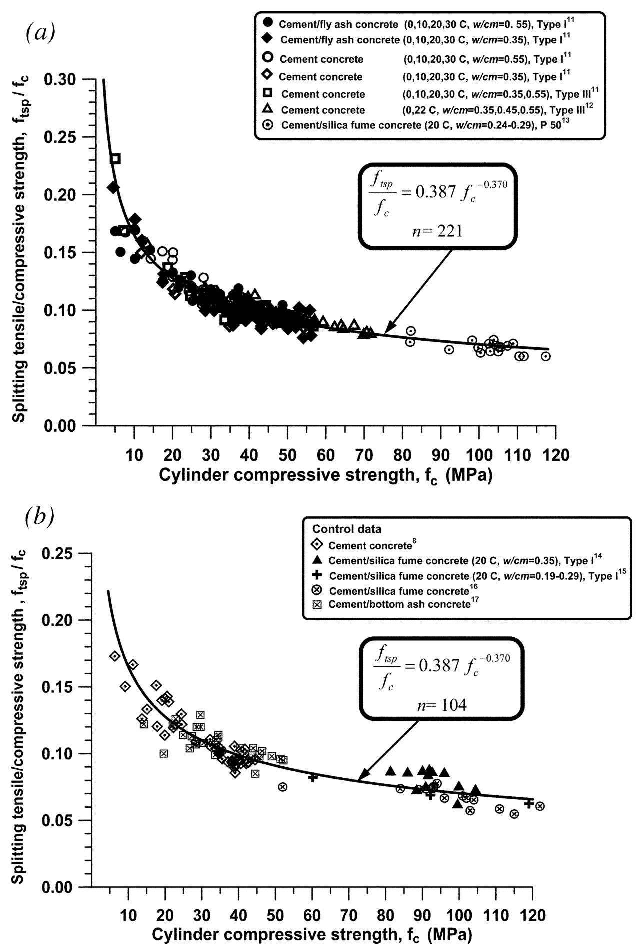


*Fig. 1—Ratio of splitting tensile to compressive strength versus cylinder compressive strength: (a) group I used to obtain A and B; and (b) compared with verification data II.*

## Discussion

As shown in Fig. 1(a), the ratio of the two strengths ($f_{tsp}/f_c$) is strongly affected by the level of the compressive strength $f_c$. This ratio decreases with increasing compressive strength at a decreasing rate. This finding can be explained by the fact that the increase in the splitting tensile strength $f_{tsp}$ occurs at a much smaller rate compared to the increase of compressive strength. The result is in agreement with various researchers.[21-24] From Fig. 1(a) and (b), it is also evident that, in comparison with normal-strength concrete (NSC), at higher strengths (80 to 120 MPa [11,600 to 17,400 psi]—very high-strength concrete) there is a significant decrease in the ratio. For example, the ratio of ($f_{tsp}/f_c$) varies between 0.15 and 0.10 for the NSC, while the same ratio is between 0.08 and 0.06 for very high-strength concrete. This finding implies that for the compressive strengths above approximately 100 MPa (14,000 psi), there is no further increase in the tensile strength.[25] According to the results published by Komloš,[24] the ratio between the splitting tensile strength and compressive strength of 200 mm cubes was found to be 0.092 and 0.067 for 7- and 180-day specimens ($w/c$ = 0.4), respectively. The examined ratio reached a value of approximately 0.06 after 360 days of curing for the same mixture. Also, the result of this study agrees with Komloš.[24]

As previously stated, there is little information in the literature concerning the accuracy and validity of the equations used for the purpose of estimating splitting tensile strength from compressive strength. This is especially true for very high-strength concretes. To assess the accuracy of other power function relationships, which are provided in Table 4, the IAE concept was used for the experimental data reported by the various researchers[11,19,20] within the range 4 to 120 MPa (580 to 17,400 psi) as well as in intervals of 20 MPa (2900 psi).

On close examination of Table 4, the following findings can be obtained:

1. Based on the values of IAE calculated, the splitting tensile strength of concrete is not proportional to the square root of compressive strength. This is particularly true for $f_c > 40$ MPa (5800 psi). The ACI models[26,27] underestimate the splitting tensile strength for concrete with compressive strength $f_c >$ 40 MPa (5800 psi). The same findings were mentioned previously by other investigators.[8,21,32]

2. In the case of the CEB-FIB equation,[10] the value of IAE varies between 2.5 and 8.9%. When all ranges are considered, the IAE is computed as 5.9%. It is interesting to note that although the equation in question is based on $f_c < 83$ MPa

**Table 4—Calculated integral absolute errors for several relationships (splitting tensile strength, cylinder compressive strength) in terms of strength range**

| Source | Relationship | IAE, % | | | | | | | Range,* MPa | Remarks |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Compressive strength, MPa (psi) | | | | | | | | |
| | | 0 to 20 (0 to 2900) | 20 to 40 (2900 to 5800) | 40 to 60 (5800 to 8700) | 60 to 80 (8700 to 11,600) | 80 to 100 (11,600 to 14,500) | 100 to 120 (14,500 to 17,400) | All ranges | | |
| ACI 363R-92[26] | $f_{tsp} = 0.59 f_c^{0.5}$ | 14.4 | 5.8 | 9.7 | 12.4 | 18.9 | 12.4 | 8.1 | $21 \le f_c \le 83$ | — |
| ACI 318-99[27] | $f_{tsp} = 0.56 f_c^{0.5}$ | 10.9 | 8.6 | 14.0 | 16.8 | 23.0 | 16.7 | — | — | — |
| CEB-FIB[10] | $f_{tsp} = 0.3 f_c^{2/3}$ | 8.9 | 6.0 | 5.6 | 2.5 | 7.9 | 8.2 | 5.9 | $f_c < 83$ | — |
| Mokhtarzadeh and French[28] | $f_{tsp} = 0.56 f_c^{0.5}$ | 10.8 | 8.6 | 14.0 | 16.8 | 23.0 | 16.7 | 18.4 | $48 \le f_c \le 103$ | For all data, moist and heat-cured |
| | $f_{tsp} = 0.32 f_c^{0.63}$ | 15.4 | 17.4 | 18.9 | 18.0 | 20.8 | 12.7 | 18.9 | | |
| Carino and Lew[29] | $f_{tsp} = 0.272 f_c^{0.71}$ | 12.4 | 8.3 | 7.1 | 3.0 | 7.8 | 8.9 | — | — | — |
| Raphael[30] | $f_{tsp} = 0.313 f_c^{0.667}$ | 10.7 | 8.9 | 9.1 | 6.5 | 9.9 | 6.2 | 9.1 | $f_c \le 40$ | Normal concrete |
| Ahmad and Shah[31] | $f_{tsp} = 0.462 f_c^{0.55}$ | 9.3 | 10.0 | 14.0 | 15.4 | 20.4 | 13.2 | 13.8 | $15 \le f_c \le 84$ | — |
| Gardner et al.[12] | $f_{tsp} = 0.47 f_c^{0.59}$ | 13.8 | 7.3 | 4.4 | 3.0 | 7.7 | 7.8 | 7.0 | $3 \le f_c \le 46$ | Type I cement concretes ($r$ = 0.865) |
| | $f_{tsp} = 0.46 f_c^{0.60}$ | 14.1 | 8.1 | 5.1 | 4.2 | 7.6 | 9.7 | 6.8 | $13 \le f_c \le 72$ | Type III cement concretes ($r$ = 0.989) |
| Gardner[11] | $f_{tsp} = 0.34 f_c^{0.66}$ | 8.8 | 5.8 | 5.4 | 2.4 | 7.9 | 8.1 | 9.7 | $4 \le f_c \le 57$ | Best-fit relationship for Type I, III cement and fly ash concrete ($r$ = 0.98, IAE = 5.8%) |
| | $f_{tsp} = 0.33 f_c^{2/3}$ | 8.9 | 6.0 | 5.6 | 2.5 | 7.9 | 8.2 | 6.0 | | Proposed relationship for Type I, III cement and cement/fly ash concrete (IAE = 6%) |
| Oluokun et al.[8] | $f_{tsp} = 0.294 f_c^{0.69}$ | 10.9 | 7.7 | 7.1 | 3.3 | 8.1 | 7.8 | 7.5 | $3.5 \le f_c \le 63$ | Normalweight concrete (IAE = 7.43%) |
| Arıoglu[32] | $f_{tsp} = 0.321 f_c^{0.661}$ | 10.0 | 8.5 | 8.8 | 6.3 | 10.0 | 6.1 | 8.5 | $15 \le f_c \le 120$ | Cement and cement/silica fume concretes ($r$ = 0.950) |
| Current study | $\frac{f_{tsp}}{f_c} = 0.387 f_c^{-0.37}$ | 9.0 | 5.6 | 4.8 | 2.3 | 8.0 | 7.4 | 5.9 | $4 \le f_c \le 120$ | Proposed for (0 to 30 °C) curing temperatures, Type I, III, cement/fly ash, cement/bottom ash, cement/silica fume concretes |

*Including control data used.
Note: 1 MPa = 145.038 psi.

(12,035 psi), it can be extrapolated to higher strengths without any loss of accuracy.

3. The equations reported by Gardner et al.,[12] Gardner,[11] and Oluokun et al.,[8] which were derived originally for normal concrete strengths, yield reasonable errors for high strengths.

4. The proposed model in the present study can be regarded as a realistic representation, which is applicable to concrete at early ages as well as to very high-strength concrete up to 120 MPa (17,400 psi) (refer to Fig. 1(a)). To further verify the proposed equation, the experimental data ($n$ = 104), which were not used in the regression analysis, compared with the regression equation, as shown in Fig. 1(b). These data are shown to be in a close agreement with Eq. (7).

## APPLICABILITY OF JOHNSTON'S STRENGTH CRITERION

Johnston[3] proposed an empirical criterion to predict the compressive strength for intact geomaterials under confinement. The criterion in question can be expressed by the following equation (Fig. 2)

$$\frac{f_1}{f_c} = \left(1 + \frac{M}{B} \cdot \frac{f_r}{f_c}\right)^B \qquad (8)$$

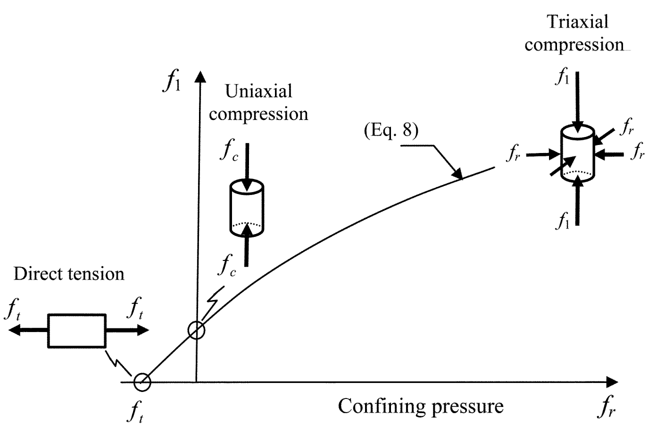


*Fig. 2—Johnston's strength criterion to estimate ultimate axial compressive strength for intact rock under triaxial compression.*

where $M$ and $B$ are the material constants; $f_1$ is ultimate compressive strength; $f_r$ is confining pressure; and $f_c$ is uniaxial compressive strength. On the basis of 1700 individual test results, this criterion was found to be applicable to a wide range of intact geomaterials varying from lightly overconsolidated clays to extremely hard rocks.

The biaxial tension condition corresponds to $f_1 = 0$ and $f_r = -f_t$, where $f_t$ is the uniaxial tensile strength (it is assumed that the biaxial tensile strength equals the uniaxial tensile strength). From this condition, the value of $M/B$ is found to be equal to the ratio of compressive to tensile strength

$$\frac{f_c}{f_t} = \frac{M}{B} \qquad (9)$$

As discussed by Johnston,[3] and as supported by limited experimental evidence,[3] this ratio is not a constant but seems to change with not only the rock type, but also the rock strength.

The material constants ($M$, $B$) in Johnston's strength criterion can be determined from a regression analysis, taking into account a series of triaxial tests on intact samples of rock or concrete. If there are no laboratory triaxial compression test data, the values of the constants can be estimated by the following two equations

$$B = 1 - 0.0172(\log 1000 f_c)^2 \qquad (10)$$

(for a wide variety fo geomaterials:[3] 0.08 MPa $\leq f_c \leq$ 600 MPa)

$$M = B\frac{f_c}{f_t} = B\frac{f_c}{\lambda f_{tsp}} \qquad (11)$$

in which $f_c$ is the uniaxial compressive strength, in MPa; and $f_{tsp}$ is the splitting tensile strength, in MPa. The value of $f_c/f_{tsp}$ can be estimated from the proposed Eq. (7) in Table 2 and Fig. 1. The value $\lambda$ is the factor for converting splitting tensile strength to direct tensile strength and is assumed to equal 0.9 (CEB-FIB[10]).

It is worthwhile to mention that the value of $B$ depends strongly on material strength. When $B$ = 1, as in the case of normally consolidated soils, Johnston's strength criterion

**Table 5—Comparison of Johnston's strength criterion with other existing strength criteria for very high-strength concretes**

| Source | Uniaxial compressive strength range $f_c$, MPa | Maximum strength ratio $f_1/f_c$ | Confinement ratio $f_r/f_c$ | Equation | No. of data | IAE*, % | IAE†, % | Remarks |
|---|---|---|---|---|---|---|---|---|
| Xie et al.[15] | 60 to 119 | 0.82 to 3.21 | 0 to 0.504 | $\frac{f_1}{f_c} = \sqrt{1 + k\left(\frac{f_r}{f_c}\right)}$; $k = 21.2 - 0.05f_c$ | 33 | 4.7 | 7.2 | Silica fume concrete $w/cm$ = 0.216 to 0.321 |
| Attard and Setunge[16] | 60 to 132 | 1.042 to 2.417 | 0.004 to 0.25 | $\frac{f_1}{f_c} = \left(1 + \frac{f_r}{f_t}\right)^k$, $(f_t = 0.9f_{tsp})$ | 24 | 3.7 | 7.9 | Silica fume concrete ($f_{tsp} = 0.62 f_c^{0.5}$) (100 to 132 MPa) |
| | | | | $k = 1.25\left[1 + 0.062\frac{f_r}{f_c}\right](f_c)^{-0.21}$ | 14 | 6.1 | | Concrete without silica fume ($f_{tsp} = 0.32 f_c^{0.67}$) (60 to 126 MPa) |

*IAE = integral absolute error for existing empirical strength equations concerning experimental triaxial compressive data by researchers reported in this table.

†IAE = integral absolute error for Johnston's strength criterion concerning experimental data reported by researchers reported in this table (the ratio of splitting tensile strength to cylinder compressive strength is predicted by proposed Eq. (7) in this study).

Note: $f_1$ = ultimate compressive strength under triaxial compression; $f_r$ = confining pressure; $k$ = constant in empirical strength criterion equation; $f_c$ = uniaxial cylinder compressive strength; $f_{tsp}$ = splitting tensile strength; $f_t$ = direct tensile strength of concrete; and 1 MPa = 145.038 psi.

simplifies to the Mohr-Coulomb criterion. In brief, $B$ defines the nonlinearity of the failure envelope (Fig. 2) and is a measure of the confinement effectiveness.

The experimental results reported by Xie et al.[15] and by Attard and Setunge[16] for high-strength concretes were compared with Johnston's strength criterion. The comparisons are displayed in Table 5. As seen from the table, the failure envelopes based on Johnston's strength criterion, for which only one parameter (cylinder compressive strength) is needed, have prediction errors (IAE) varying between 7.2 and 7.9%. Such an error level can be regarded acceptable for practical engineering applications. The failure envelopes given by the various researchers[15,16] result in smaller prediction errors (IAE = 3.7 to 6.1%) because these failure expressions were based on the specific experimental data.

Figure 3 shows a comparison of the computed axial compressive strengths $f_1$ from Johnston's strength criterion and two triaxial compression data sets[15,16] (the comparison was done in terms of the normalized strengths: $f_1/f_c$; $f_r/f_c$). From Fig. 3, it can be seen that the estimates obtained from Johnston's strength criterion for high-strength concretes are in good agreement with the failure envelopes obtained by the researchers.[15,16] The linear failure envelope ($f_1 = f_c + 4.1f_r$) proposed by Richart et al.[33] underestimates the triaxial compressive concrete because their equation was based on tests of low-strength concrete ($f_c$ = 32 MPa).

In the following section, a numerical example will demonstrate how to use Johnston's strength criterion for evaluation of the strength of high-strength concrete under triaxial compression.

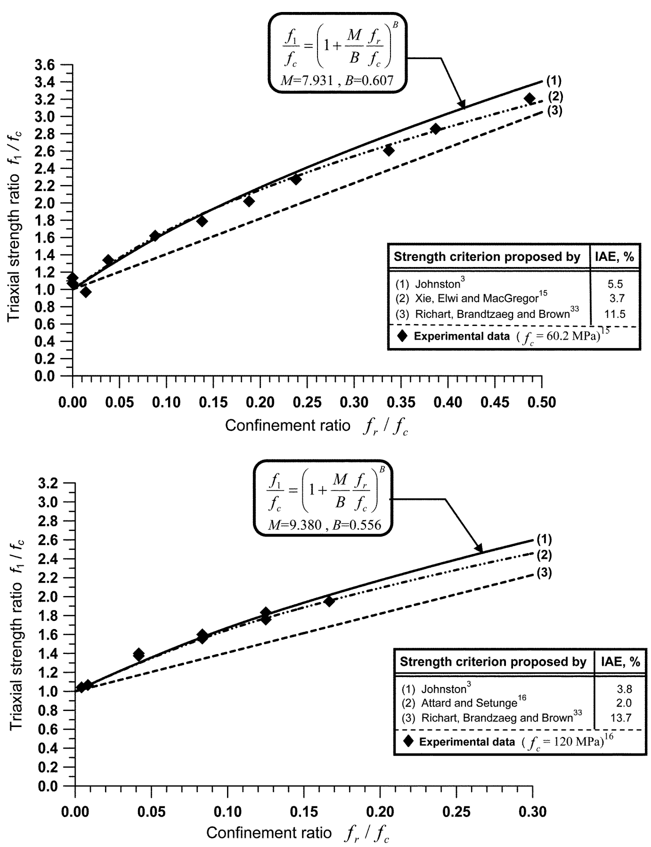


*Fig. 3—Experimental data of concretes under triaxial compression compared with strength criteria proposed by various researchers.*

## NUMERICAL EXAMPLE

The following experimental results were obtained from a study[16] carried out on concrete subjected to triaxial compression.

- Cylinder compressive strength: $f_c$ = 60 MPa (8700 psi) ($w/cm$ = 0.45).
- Applied confining pressure: $f_r$ = 10 MPa (1450 psi).

The following steps are used to compute the ultimate axial compressive strength $f_1$ and the benefit of confinement for the given confining pressure $f_r$:

- Estimate the splitting tensile strength to cylinder compressive strength ratio:

From Eq. (7): $f_{tsp}/f_c = 0.387f_c^{-0.37} = 0.387 \times 60^{-0.37} = 0.085$.

- Estimate the material constants ($M$, $B$) corresponding to Johnston's strength criterion:

$$B = 1 - 0.0172(\log 1000 f_c)^2 = 1 - 0.0172(\log 1000 \times 60)^2 = 0.607$$

$$f_t = \lambda \cdot f_{tsp}$$

$$\frac{M}{B} = \frac{f_c}{f_t} = \frac{f_c}{0.9 f_{tsp}} \rightarrow M = \frac{0.607}{0.9 \times 0.085} = 7.93$$

- Estimate the failure envelope under triaxial compression:

$$\frac{f_1}{f_c} = \left(1 + \frac{M}{B} \cdot \frac{f_r}{f_c}\right)^B = \left(1 + \frac{7.93}{0.607} \cdot \frac{10}{60}\right)^{0.607} = 2.017$$

$$f_1 = 2.017 f_c = 2.017 \cdot 60 = 121.0 \text{ MPa (17,550 psi)}$$

- Evaluation of the above result:

According to Attard and Setunge's study,[16] the axial compressive strength $f_1$ was determined to be 122 MPa (17,694 psi) (p. 435, Table 3) for the given confining pressure. Thus, the calculated ultimate compressive strength is in excellent agreement with the experimental finding.

For $f_r$ = 10 MPa (1450 psi), the benefit of confinement can be evaluated by means of the efficiency ratio:

$$\text{efficiency ratio (\%)} = \frac{f_1}{f_c} \cdot 100 = \frac{121}{60} \cdot 100 \cong 200\%$$

As can be seen, the axial compressive strength $f_1$ increases two-fold due to a modest confinement stress. Moreover, for a given confining pressure $f_r$, the equations published by Balmer,[34] in which normal and shear stresses are related to principal stresses, can be used[5,35] to estimate the angle of friction $\phi$, the failure angle $\alpha$, and the cohesive strength $C$.

## CONCLUSIONS

The following conclusions can be drawn from this study:

1. A simple power function is proposed to evaluate the ratio of the splitting tensile to compressive strength ($f_{tsp}/f_c$) as a function of the cylinder compressive strength $f_c$ (Table 2, Eq. (7), Fig. 1(a)). Based on the error analysis (Table 3), Eq. (7) is reasonably accurate and is applicable to concrete strengths ranging from 4 to 120 MPa, (580 to 17,400 psi) regardless of mixture proportions, the nature of the cementitious materials, curing time, and curing temperature;

2. The ratio of the tensile strength to compressive strength ($f_{tsp}/f_c$) is influenced by the level of concrete strength. At low compressive strengths, the splitting tensile strengths are as high as 10% of the cylinder compressive strength but at extremely higher compressive strengths, the ratio reduces to approximately 5% (Fig. 1);

3. The commonly accepted 0.5 power relationship between the splitting tensile strength and cylinder compressive strength was not determined to be realistic; thus the ACI model should be re-evaluated for high-strength concrete; and

4. It is demonstrated that, knowing only the cylinder compressive strength of concrete, Johnston's strength criterion (Eq. (8)) and the ratio of the splitting tensile to compressive strength (Eq. (7)) can be used to estimate the axial compressive strength of concrete under confining stress without performing triaxial tests (Table 5 and Fig. 3).

## ACKNOWLEDGMENTS

The authors are greatly indebted to the members of the board, Yapı Merkezi Inc., Istanbul, Turkey, for their close attention and encouragement throughout the course of this study. E. Arıoglu, Honorary Chair of Yapı Merkezi Inc., is gratefully acknowledged for his academic interest and support. The assistance of Ö. S. Köylüoglu is also appreciated.

## REFERENCES

1. Bortolotti, L., "First Cracking Load of Concrete Subjected to Direct Tension," *ACI Materials Journal*, V. 88, No. 1, Jan.-Feb. 1991, pp. 70-73.

2. Bortolotti, L., "Influence of Concrete Tensile Ductility on Compressive Strength of Confined Columns," *Journal of Materials in Civil Engineering*, ASCE, V. 6, No. 4, Nov. 1994, pp. 542-563.

3. Johnston, I. W., "Strength of Intact Geomechanical Materials," *Journal of Geotechnical Engineering*, ASCE, V. 111, No. 6, June 1985, pp. 730-748.

4. Setunge, S.; Attard, M. M.; and Darvall, P. P., "Ultimate Strength of Confined Very High-Strength Concretes," *ACI Structural Journal*, V. 90, No. 6, Nov.-Dec. 1993, pp. 632-641.

5. Yapı Merkezi Inc., "Investigation of the Failure Criteria for Concrete in Triaxial Compression, and Determination of its Material Properties such as Angle of Internal Friction, Cohesion without Making any Experiments," *Yapı Merkezi Internal Research Report*, Report No. YM/ARGE/96-12, July 1996, Istanbul, 27 pp. (in Turkish)

6. Neville A. M., *Properties of Concrete*, 4th Edition, Longman Group Ltd., Essex, 1995, 844 pp.

7. Hannant, D. J.; Buckley, K. J.; and Croft, J., "The Effect of Aggregate Size on the Use of the Cylinder Splitting Test as a Measure of Tensile Strength," *Materials and Structures*, V. 6, No. 31, 1973, pp. 15-21.

8. Oluokun, F. A.; Burdette, E. G.; and Deatherage, J. H., "Splitting Tensile Strength and Compressive Strength Relationships at Early Ages," *ACI Materials Journal*, V. 88, No. 2, Mar.-Apr. 1991, pp. 115-121.

9. Kadlecek, V.; Modry, S.; and Kadlecek, V. J., "Size Effect of Test Specimens on Tensile Splitting Strength of Concrete: General Relation," *Materials and Structures*, V. 35, Jan.-Feb. 2002, pp. 28-34.

10. CEB-FIP Model Code for Concrete Structures 1990, "Evaluation of the Time Dependent Behaviour of Concrete," *Bulletin d'Information No. 199*, Comite European du Béton/Fédération Internationale de la Precontrainte, Lausanne, 1991, 201 pp.

11. Gardner, N. J., "Effect of Temperature on the Early-Age Properties of Type I, Type III, and Type I/Fly Ash Concretes," *ACI Materials Journal*, V. 87, No. 1, Jan.-Feb. 1990, pp. 68-78.

12. Gardner, N. J.; Sau, P. L.; and Cheung, M. S., "Strength Development and Durability of Concretes Cast and Cured at 0 C," *ACI Materials Journal*, V. 85, No. 6, Nov.-Dec. 1988, pp. 529-536.

13. Imam, M.; Vandewalle, L.; and Mortelmans, F., "Indirect Tensile Strength of Very High Strength Concrete," *Proceedings of 5th International Symposium on Utilization of High Strength/High Performance Concrete,* V. 2, Sandefjord, Norway, 1999, pp. 1114-1121.

14. Irvine, H., and Montgomery, F. R., "Optimising the Use of the Coarse Aggregates of the North of Ireland in the Production of High Strength Concrete," *Proceedings of 5th International Symposium on Utilization of High Strength/High Performance Concrete*, V. 2, Sandefjord, Norway, 1999, pp.1164-1173.

15. Xie, J.; Elwi, A. E.; and MacGregor, J. G., "Mechanical Properties of Three High-Strength Concretes Containing Silica Fume," *ACI Materials Journal*, V. 92, No. 2, Mar.-Apr. 1995, pp. 135-145.

16. Attard, M. M., and Setunge, S., "Stress-Strain Relationship of Confined and Unconfined Concrete," *ACI Materials Journal*, V. 93, No. 5, Sept.-Oct. 1996, pp. 432-442.

17. Ghafoori, N., and Bucholc, J., "Properties of High-Calcium Dry Bottom Ash Concrete," *ACI Materials Journal*, V. 94, No. 2, Mar.-Apr. 1997, pp. 90-101.

18. Dudewicz, E. J., "Basic Statistical Methods," *Juran's Quality Control Handbook*, 4th Edition, J. M. Juran and F. M. Gryna, eds., McGraw-Hill International Editions, Section 23, New York, 1988, pp. 1-121.

19. Oluokun, F. A., "Prediction of Concrete Tensile Strength from its Compressive Strength Evaluation Relations for Normal Weight Concrete," *ACI Materials Journal*, V. 88, No. 3, May-June 1991, pp. 302-309.

20. Arıoglu, E., Discussion of "Effects of Size and Curing on Cylinder Compressive Strength of Normal and High-Strength Concretes," by P. C. Aïtcin, *ACI Materials Journal*, V. 92, No. 3, May-June 1995, pp. 332-334.

21. Zain, M. F. M.; Mahmud, H. B.; Ilham, A.; and Faizal, M., "Prediction of Splitting Tensile of High-Performance Concrete," *Cement and Concrete Research,* V. 32, 2002, pp. 1251-1258.

22. Arıoglu, E.; Girgin, C.; and Arıoglu, N., "Re-Evaluation of Ratio of Tensile Strength to Compressive Strength for Normal-Strength Concrete," *Journal of Ready Mix Concrete*, Jan.-Feb. 2002, pp. 58-63. (in Turkish)

23. Li, Q., and Ansari, F., "High-Strength Concrete in Uniaxial Tension," *ACI Materials Journal*, V. 97, No. 1, Jan.-Feb. 2000, pp. 49-57.

24. Komloš, K., "Comments on the Long-Term Tensile Strength of Plain Concrete," *Magazine of Concrete Research*, V. 22, No. 73, Dec. 1970, pp. 232-238.

25. König, G., "High Strength Concrete," *Darmstadt Concrete*, V. 6, 1991, pp. 95-115.

26. ACI Committee 363, "State-of-the-Art Report on High-Strength Concrete (ACI 363R-92)," American Concrete Institute, Farmington Hills, Mich., 1992, 55 pp.

27. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-99) and Commentary (318R-99)," American Concrete Institute, Farmington Hills, Mich., 1999, 391 pp.

28. Mokhtarzadeh, A., and French, C., "Mechanical Properties of High-Strength Concrete with Consideration for Precast Applications," *ACI Materials Journal*, V. 97, No. 2, Mar.-Apr. 2000, pp. 136-147.

29. Carino, N. J., and Lew, H. S., "Re-Examination of the Relation Between Splitting Tensile and Compressive Strength of Normal Weight Concrete," ACI JOURNAL, *Proceedings* V. 79, No. 3, May-June 1982, pp. 214-219.

30. Raphael, J. M., "Tensile Strength of Concrete," ACI JOURNAL, *Proceedings* V. 81, No. 2, Mar.-Apr. 1984, pp. 158-165.

31. Ahmad, S. H., and Shah, S. P., "Structural Properties of High Strength Concrete and its Implications for Precast Prestressed Concrete," *PCI Journal*, V. 30, No. 6, Nov.-Dec. 1985, pp. 92-119.

32. Arıoglu, E., Discussion of "Relationship Between Splitting Tensile Strength and the Compressive Strength" by V. Yerlici and U. Ersoy, *IMO Technical Journal*, No. 4, Oct. 1995, pp. 1059-1062. (in Turkish)

33. Richart, F. E.; Brandtzaeg, A.; and Brown, R. L., "Failure of Plain and Spirally Reinforced Concrete in Compression," *Bulletin 190*, University of Illinois, Engineering Experimental Station, Champaign, Ill., 1929.

34. Balmer, G., "A General Analytical Solution for Mohr's Envelope," *Proceedings*, ASTM International, V. 52, 1952, pp. 1260-1271.

35. Arıoglu, E.; Girgin, C.; and Arıoglu, N., "Estimation of Shear Parameters for Very High-Strength from Strength Criteria," *Concrete Prefabrication*, No. 71, July 2004, pp. 5-17. (in Turkish)

Disc. 103-M03/From the Jan.-Feb. 2006 *ACI Materials Journal*, p. 18

**Evaluation of Ratio between Splitting Tensile Strength and Compressive Strength for Concretes up to 120 MPa and its Application in Strength Criterion.** Paper by Nihal Arıoglu, Z. Canan Girgin, and Ergin Arıoglu

**Discussion by Nabi Yüzer**

*Associate Professor of Civil Engineering, Yildiz Technical University, Istanbul, Turkey.*

The authors' evaluations on the ratio of splitting tensile strength to cylinder compressive strength of concrete as a function of compressive strength in a large scale are appreciated. In this study, it is noted that Eq. (7) is applicable to concrete with various mixture porportions, cementitious materials with and without silica fume and fly ash, and various curing times and curing temperatures.[36]

$$\frac{f_{tsp}}{f_c} = 0.387 f_c^{-0.370} \qquad (7)$$

It has recently been shown, however, that the equations for determining splitting tensile strength as a function of compressive strength lose their validity in the condition of external or internal diffusion of chloride to concrete.[37,38] It is also shown in Fig. A that Eq. (7) proposed by the authors is not valid for the 72 concrete samples with and without silica fume,[39] and 60 concrete samples with and without ground granulated blast-furnace slag under chloride effect.[40]

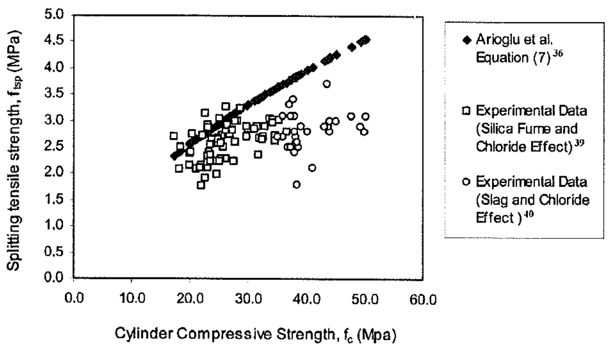


*Fig. A—Relation between tensile and compressive strength of concrete exposed to chloride.*

In other words, the discusser put the compressive strength values of 132 concrete samples in total into Eq. (7) to predict splitting tensile strength values. It could be seen in Fig. A, however, that experimentally measured splitting tensile strength values (which were published in References 39 and 40) deviate dramatically from the predicted ones found by using Eq. (7) recommended by the authors.

Consequently, in evaluating the quality of concrete and/or reinforced concrete exposed to chloride effect, the empirical equations for the tensile strength consisting only of the compressive strength as a material parameter would not be valid; the tensile strength should be tested separately.

## NOTATION

$f_c$ = cylinder compressive strength
$f_{tsp}$ = splitting tensile strength

## REFERENCES

36. Arıoglu, N.; Girgin, Z. C.; and Arıoglu, E., "Evaluation of Ratio between Splitting Tensile Strength and Compressive Strength for Concretes up to 120 MPa and its Application in Strength Criterion," *ACI Materials Journal*, V. 103, No. 1, Jan.-Feb. 2006, pp. 18-24.

37. Yüzer, N., and Aköz, F., "The Relation Between Tensile and Compressive Strength of Concrete Exposed to Chloride," *Technical Journal*, Digest, Turkish Chamber of Civil Engineers, V. 16, 2005, p. 1050.

38. Yüzer, N., and Aköz, F., "The Relation Between Tensile and Compressive Strength of Concrete Exposed to Chloride," *Technical Journal*, Turkish Chamber of Civil Engineers, V. 16, No. 4, Oct. 2005, pp. 3673-3681. (in Turkish)

39. Yüzer, N., "The Investigation of Chloride Effect on Reinforced Concrete Members with Silica Fume by Using Accelerated Corrosion Test," PhD thesis, Yildiz Technical University, Istanbul, Turkey, 1998.

40. Cakir, Ö., "Effect of Ground Granulated Blast-Furnace Slag on the Durability of Concrete and Reinforcement," PhD thesis, Yildiz Technical University, Istanbul, Turkey, 2006. (in Turkish)

Disc. 103-M03/From the Jan.-Feb. 2006 *ACI Materials Journal*, p. 18

**Evaluation of Ratio between Splitting Tensile Strength and Compressive Strength for Concretes up to 120 MPa and its Application in Strength Criterion.** Paper by Nihal Arıoglu, Z. Canan Girgin, and Ergin Arıoglu

**Discussion by Kenneth W. Day**

*FACI, Consulting Concrete Technologist, Melbourne, Australia.*

The discusser would anticipate that the ratio of splitting to compressive strength would be substantially dependent on coarse aggregate particle shape. The paper, however, gives no information as to whether this has been considered.

Could the authors please comment?

## AUTHORS' CLOSURE

The authors would like to thank the discussers: Yüzer for his stimulating remarks and Day for his keen interest in the paper.

### Reply to Nabi Yüzer

The purpose of the research was to derive a reliable relationship between the ratio of splitting tensile strength to cylinder compressive strength and the compressive strength for concretes up to 120 MPa. Moreover, the relationship was used to verify Johnston's strength criterion in intact rocks for the ultimate strength of very high-strength concrete under triaxial compression. The authors entirely agree with the discusser that, in the case of concretes subjected to effect of

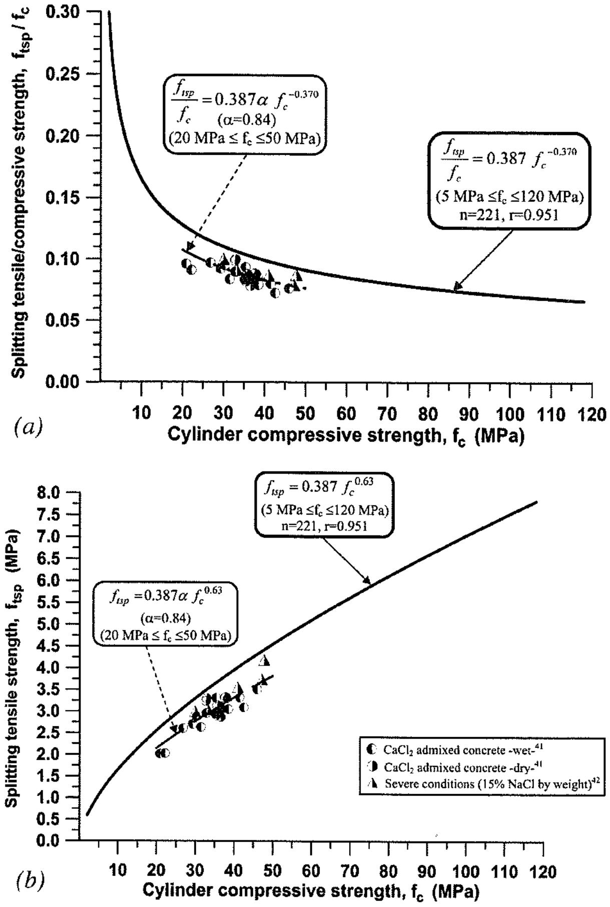


*Fig. B—Modified relationships between tensile (ϕ150 x 300 mm) and compressive (ϕ150 x 300 mm) strength of concrete subjected to chloride effect* (n = *number of data used in regression analysis;* r = *coefficient of correlation, and* $\alpha_c$ = *calibration factor included chloride effect).*

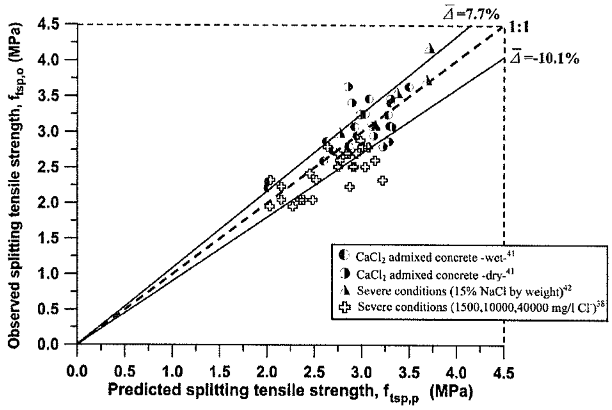


*Fig. C—Comparison between experimental data gathered from various sources and these predicted values from modified Eq. (7)*[36] *(deviation* $\Delta = (f_{tsp,o} - f_{tsp,p})/f_{tsp,o} \times 100$, *%.*

chloride ion, use of any of the relationships in which the splitting strength is estimated on the basis of the compressive strength will considerably overestimate the splitting strength of concrete. Recalling the study of Rezansoff and Corbett,[41] a chloride-based setting and accelerating admixture reduces the splitting tensile strength as compared with normal—no accelerated—concrete. In this discussion, our equation, Eq. (7),[36] introduced a modification factor $\alpha_c$ to account for the effect of chloride on the tensile strength of concrete as displayed in Fig. B. The value of α was obtained by means of least-squares regression analysis. For this analysis, the data were gathered from Rezansoff and Corbett's[41] and Issa et al.'s[42] studies. According to the analysis, the statistical values of $\alpha_c$ were determined as the average $\bar{\alpha}_c = 0.84$, the standard deviation $S = 0.05$, and the coefficient of variation $V = S/\bar{\alpha}_c \times 100 = 5.9\%$. From the value of $V$, it can be concluded that there is a very limited variation in the value of $\alpha_c$. In other words, the mean $\bar{\alpha}_c$ of 0.84 can be taken as "constant of proportionality" for Eq. (7) correlating the splitting tensile strength with the compressive strength of concrete subjected to chloride effect.

In passing, it should be mentioned that the influence of strength range, curing condition, time, and concentration of chloride ion on the value of $\alpha_c$ are still unknown. In brief, for the assessment of the influences in question, the detailed experimental test program and statistical analysis are required.

To evaluate the accuracy of the modified Eq. (7) against experimental results, the data (Table A) reported by Yüzer and Aköz[38] were used as control data. The splitting tensile strengths predicted from the equation under review are compared with the experimental data in Fig. C by making use of the 1:1 technique. The average derivations ($\bar{\Delta} = -10.1\%$; $\bar{\Delta} = +7.7\%$) obtained can be considered to be acceptable.

## Table A—Compressive and splitting tensile strengths of concrete subjected to chloride effect[38]

| Specimen code | $f_c$ | $f_{tsp}$ | $f_c$ | $f_{tsp}$ | $f_c$ | $f_{tsp}$ |
|---|---|---|---|---|---|---|
| | Curing time | | | | | |
| | 28 + 28 days | | 28 + 90 days | | 28 + 180 days | |
| SF0W0 | 26.4 | 2.2 | 27.8 | 2.7 | 29.4 | 2.7 |
| SF1W0 | 26.3 | 2.6 | 26.1 | 2.9 | 32.8 | 2.9 |
| SF2W0 | 35.1 | 2.6 | 37.6 | 2.9 | 41.0 | 2.9 |
| SF0W1 | 18.3 | 2.1 | 20.1 | 2.2 | 23.1 | 2.2 |
| SF1W1 | 32.6 | 2.7 | 24.7 | 2.6 | 27.8 | 3.0 |
| SF2W1 | 36.6 | 2.8 | 32.4 | 2.7 | 31.8 | 2.4 |
| SF0W2 | 25.2 | 2.2 | 23.6 | 2.2 | 25.8 | 2.5 |
| SF1W2 | 29.7 | 2.7 | 33.8 | 3.1 | 35.3 | 3.0 |
| SF2W2 | 31.6 | 2.9 | 33.0 | 2.9 | 38.1 | 2.5 |
| SF0W3 | 21.9 | 2.1 | 20.1 | 2.4 | 18.4 | 2.5 |
| SF1W3 | 30.0 | 2.8 | 34.1 | 3.0 | 34.7 | 2.7 |
| SF2W3 | 31.2 | 2.9 | 34.2 | 3.0 | 29.7 | 2.9 |

Notes: SF0 equals mixtures without silica fume; SF1,2 equal mixtures with 10%+ and 20% silica fume by weight of concrete; W0 equals specimens kept in only water (20 ± 3 °C); W1,2,3 equal specimens kept in water containing various $Cl^-$ chloride ion concentration (1500 mg/L, 10,000 mg/L, and 40,000 mg/L); and W3 equals solution prepared at rate of 40,000 mg/L chloride ion concentration.
The size of specimens used in compressive and splitting tension tests were 100 x 200 mm (4 x 8 in.) cylinders. (The value of 0.934[43,44] as conversion factor of splitting tensile strength determined on ϕ100 x 200 cylinder specimen to strength of ϕ150 x 300 cylinder specimen was applied.)

## Reply to Kenneth W. Day

It is well known from the literature that the aggregate characteristics (type, particle shape, surface texture) have an important effect on the mechanical properties.[45,46] In particular, the bond between the aggregate and the paste is a key factor affecting the tensile strength of concrete. This mechanical interlocking, or bond strength, can be enhanced by using clean aggregates with angular shape and rough texture.[47] Hence, the tensile strength of concrete can be increased at a

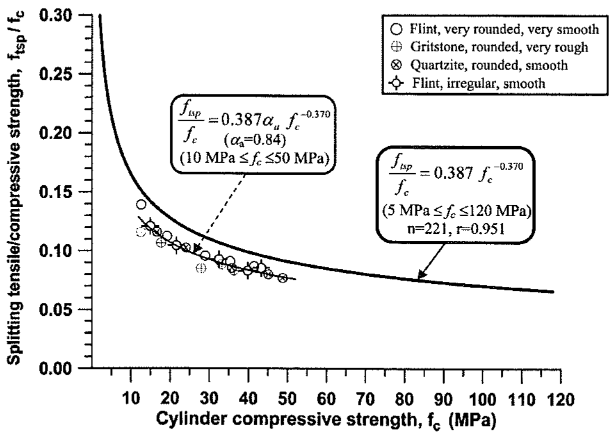


*Fig. D—Modified relationships between ratio of splitting tensile (ϕ150 x 300 mm) to compressive strength (ϕ150 x 300 mm) and cylinder compressive strength for concretes made with uncrushed gravel aggregates ($\alpha_c$ = calibration factor taking aggregate shape and texture into consideration.)*

given water-cement ratio. As far as the compressive strength is concerned, this mechanical property is less affected by the variation in aggregate characteristics[45] compared with the splitting tensile strength.

The ratio between splitting tensile strength and compressive strength $f_{tsp}/f_c$ is likely to be affected by aggregate properties in question. The majority of data used in our regression analysis belongs to the concretes made with crushed aggregates and the resulting equation, Eq. (7),[36] is valid for the concretes made from these aggregates.

To quantitatively determine the effect of aggregate with round shape and smooth texture on the ratio of $f_{tsp}/f_c$, an additional statistical analysis (Fig. D) was performed through Franklin and King's experimental data[45] (the concretes made with a constant water-cement ratio of 0.50, aggregate of rounded/irregular shape and smooth texture—uncrushed gravel—maximum aggregate size $D_{max}$ = 19 mm, various curing ages, the compressive strengths of 150 mm cube were converted to the strength of ϕ150 x 300 mm cylinder via the conversion factor of 0.8[48]). From Fig. D it is evident that, for a given compressive strength (10 MPa ≤ $f_c$ ≤ 50 MPa), the ratio of $f_{tsp}/f_c$ for concretes with uncrushed gravels results in a reduction of almost 16% compared to the concretes containing crushed rock aggregates. Briefly, in the concretes with uncrushed gravel for a given compressive strength, there is a decrease in the splitting tensile strength. Its main reason is the weaker bond developed between the aggregate and the surrounding hydrated cement paste.

## REFERENCES


41. Rezansoff, T., and Corbett, J. R., "Influence of Accelerating Admixtures on Strength Development of Concrete under Wet and Dry Curing," *ACI Materials Journal*, V. 85, No. 6, Nov.-Dec. 1988, pp. 519-528.

42. Issa, M. A.; Issa, M. A.; Faraj, M.; and Reddy, K., "Mechanical Properties and Durability of High-Performance Protecrete Concrete," *Progress Report* by University of Illinois submitted to Applied Concrete Technology, Feb. 1999, 39 pp.

43. Torrent, R. J., and Brooks, J. J., "Application of the Highly Stressed Volume Approach to Correlated Results From Different Tensile Tests of Concrete," *Magazine of Concrete Research*, V. 37, No. 132, Sept. 1985, pp. 175-184.

44. Kadlecek, V.; Modry, S. S.; and Kadlecek, J., "Size Effect of Test Specimens on Tensile Splitting Strength of Concrete: General Relation," *Materials and Structures*, V. 35, Jan.-Feb. 2002, pp. 28-34.

45. Franklin, R. E., and King, T. M., "Relations Between Compressive and Indirect Tensile Strengths of Concrete," *RRL Report* LR 412, Road Research Laboratory, UK, 1971.

46. Arıoglu, E.; Girgin, Z. C.; and Arıoglu, N., "Evaluation of the Ratio of Splitting Tensile Strength to Compressive Strength in Concrete," *Ready Mixed Concrete Magazine*, Jan.-Feb. 2002, pp. 58-63. (in Turkish)

47. Hannant, D. J., "The Tensile Strength of Concrete: A Review Paper," *The Structural Engineer*, V. 50, No.7, July 1972, pp. 253-258.

48. Arıoglu, E.; Arıoglu, N.; and Girgin, Z. C., "Shape-Size Effect in Normal and High-Strength Concrete Samples," *Ready Mixed Concrete Magazine*, Jan.-Feb. 1999, pp. 40-51. (in Turkish)


---

Disc. 103-M08/From the Jan.-Feb. 2006 *ACI Materials Journal*, p. 60

## Influence of Concrete Material Ductility on Shear Response of Stud Connections. Paper by Shunzhi Qian and Victor C. Li

### Discussion by Shiming Chen

*Professor, School of Civil Engineering, Tongji University, Shanghai, China.*

The discusser appreciates the authors' comprehensive work to investigate the potential application of engineered cementitious composite (ECC) in shear stud connections for steel-concrete composite beams with a desired ductile slip capacity. Some test findings that are interesting to the discusser, however, were not well clarified when the test results were compared with the predictions based on the design method (AASHTO LRFD). Discussions are drawn as follows:

### Compressive strength of concrete

Accordingly, it is understood that $f_c'$, the compressive strength of concrete in Table 2, should be the cylinder compressive strength. For a comparison, the measured and predicted strength per stud based on AASHTO equations were drawn in Fig. A.

It is found that the predicted strengths based on AASHTO LRFD method were all greater than the measured strengths for concrete and RC stud connections so that the method would be unsafe, which is argued by the discusser. As being noted that a steel reinforcement ratio of 0.86% had been used for transverse reinforcement in RC specimens that could prevent earlier longitudinal splitting shear failure in the connections, is $f_c'$ adopted in the paper a cube compressive strength rather than the cylinder compressive strength?

### Strength and failure modes

In design practice, the shear stud connections are normally classified as ductile as far as ratio $h/d > 4$, where $h$ and $d$ are the overall height and diameter of a stud, respectively. Ductile behaviors were observed in RC, SFRC, and ECC specimens, except for concrete specimens.