**Stress-Strain Relationship of Confined and Unconfined Concrete**

Paper by M.M. Attard and S. Setunge

Discussion by Ergin Arıoğlu


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Application of high strength concrete (HSC) technology is growing in the construction sector at an increasing rate due to the advantages on mechanical properties and durability of concrete. This issue calls for identification of the mechanical properties of HSC and thus establishment of the design equations for HSC. The authors' contributions to this aim are outstanding and they should be acknowledged herein for their excellent article.

The discusser, being stimulated from the authors' study on aggregate effect, would like to contribute on two subjects: First, an equation is presented to determine modulus of elasticity of concrete based on mechanical properties of the aggregate. Second, the strain of concrete at peak stress, as determined by Attard-Setunge's formula is compared with the discusser's earlier study.

- Modulus of elasticity of high strength of concrete is directly related to mechanical properties of the aggregate. In the recent literature, modulus of elasticity of concrete is proposed as in the following[32]:

$$\mathbf{E_c = 8430\, \alpha_\beta\, f_c'^{\,0.333}} \quad \mathbf{(10\ MPa \le f_c' \le 115\ MPa)} \qquad \text{(A-1)}$$

where

$$\alpha_\beta = \mathbf{0.1485\sqrt{E_a}} \quad \mathbf{(10\ GPa \le E_a \le 115\ GPa)} \qquad \text{(A-2)}$$

However, the latter expression can be easily expressed in terms of the uniaxial strength of intact rock, which is a more convenient property to measure in a common laboratory. The following expression by Arıoğlu [33] to determine modulus of elasticity of rock from its uniaxial strength, is based upon N=467 samples of rock including metamorphic, volcanic and sedimentary rocks and has a very high correlation coefficient.

$$\mathbf{E_a = 0.41\, f_a^{0.93}} \quad \mathbf{(N=467,\ r=0.804,\ 15\ MPa \le f_a \le 400\ MPa)} \qquad \text{(A-3)}$$

Substituting Eqs.(A-1) and (A-2) into Eq.(A-3) one obtains:

$$\mathbf{E_c = 801.578 f_a^{0.465} f_c'^{\,0.333}} \qquad \text{(A-4)}$$

In equations (A-1) to (A-4), $\mathbf{E_a}$ is the modulus of elasticity of intact rock (aggregate) [GPa], $\mathbf{E_c}$ is the modulus of elasticity of concrete [MPa], $f_a$ is the uniaxial compressive strength of intact rock [MPa], $\mathbf{f'_c}$ is the uniaxial compressive strength of concrete [MPa] and $\alpha_\beta$ is a coefficient related to the rock (aggregate) type. For example, for basalt having $\mathbf{f_a}$=200 MPa, $\alpha_\beta$ may be estimated to be about 1.12.

For a range of concrete strength from normal strength to very high strength, the modulus of elasticity values obtained from Eq.(A-5) are compared with Carasquillo et al.'s[26] formula (Eq.5) for air dry unit weight of concrete at time of test $\rho$=2320 kg/m$^3$ (Fig.A.). The following conclusions can be derived from Fig.A. :

$$\mathbf{E_c} = \left(\mathbf{3320}\sqrt{\mathbf{f_c'}} + \mathbf{6900}\right)\left(\frac{\rho}{\mathbf{2320}}\right)^{\mathbf{1.5}} \tag{A-5}$$

In high-strength concrete, modulus of elasticity of concrete is very sensitive to the uniaxial compressive strength of aggregate. Carasquillo et. al.'s equation falls within the range of aggregate strength widely used in structural applications and confirms its validity. However, their formula defines a minimum aggregate strength value of 150 MPa. This outcome points out an efficient control in selection of the aggregate for HSC.

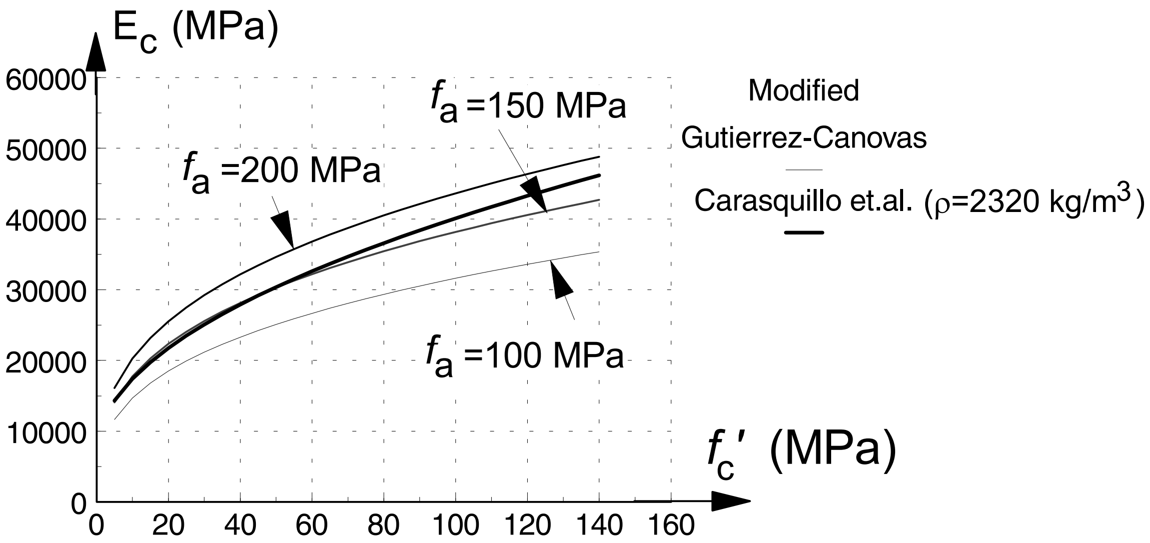


Fig.A. Comparison of Carasquillo et.al.'s Formula with Eq (A-1) through (A-3)

- The equation proposed by Attard and Setunge for prediction of strain of concrete at peak compressive stress ($\varepsilon_c$, %o) is compared with Arıoğlu's equation[33,34] (Eq. [A-6]) and it is found that the two are in close agreement with each other (Fig.B[a]). The equation by Arıoğlu[33] can additionally take into account the volume of the specimen (Eq.[A-6]-Fig.B[b]). The error involved in prediction capacity of the equations are evaluated based on deviation ($\Delta$) of the predicted values $\varepsilon_{c,p}$ from the measured $\varepsilon_{c,m}$ and, additionally, based on the integral absolute error (IAE) concept[33] (Fig.C.). Setunge's equation is only compared with the tests on 100x200 mm cylinder specimens whereas Arıoğlu's equation can be used with any specimen size. Average deviation for Setunge's equation is approximately $\overline{\Delta} = \pm 6.8$ percent and for Arıoğlu's equation it is estimated approximately $\overline{\Delta} = \pm 6.6$ percent (Fig.C.). The IAE values for the two equations are, 6.3 and 6.1 percent, respectively.

$$\varepsilon_{\mathbf{c}} = \mathbf{1.753}\ \mathbf{f_c'}^{\mathbf{0.27756}}\ \mathbf{V}^{\mathbf{-0.09314}} \tag{A-6}$$

in which, $\varepsilon_{\mathbf{c}}$ is the strain at peak stress (i.e. compressive strength)[%o], $\mathbf{f_c'}$ is the cylinder compressive strength [MPa], V is the volume of the concrete test cylinder [$cm^3$].

Another analysis by the discussers is aimed at determination of the sensitivity of the $\varepsilon_c$ , with respect to the aggregate type (Fig.D). When Setunge's equation is utilized with $E_c$ from Carasquillo et.al., the results are reasonably close to those calculated by the same $\varepsilon_c$ equation with $E_c$ from Eq.(A-4) for aggregate strength between 150 and 200 MPa. It is also conceivable to obtain that for $\mathbf{f_c'} \leq 20$ MPa, with $\mathbf{f_a}$=100 MPa, $\varepsilon_{\mathbf{c}} = 0.002$. With increasing strength $\varepsilon_{\mathbf{c}}$ increases. For instance, for 80 MPa concrete with high strength aggregate (100-150 MPa), $\varepsilon_c$ varies between ~ 0.0028-0.0032.

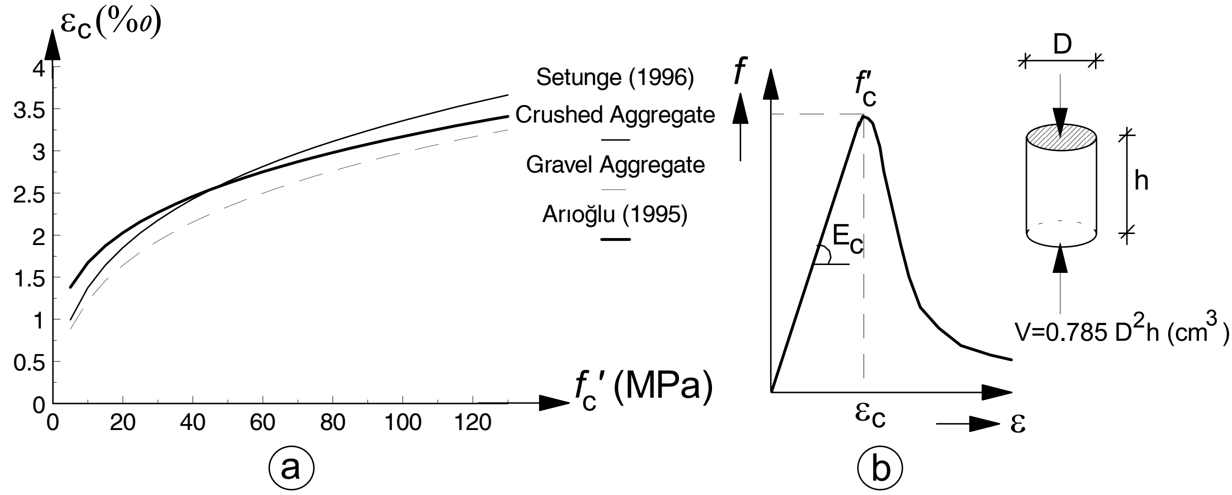


Fig. B- Comparison of Setunge's Equations ($E_c$ from Carasquillo et.al.) with Equation Proposed by Arıoğlu[33]

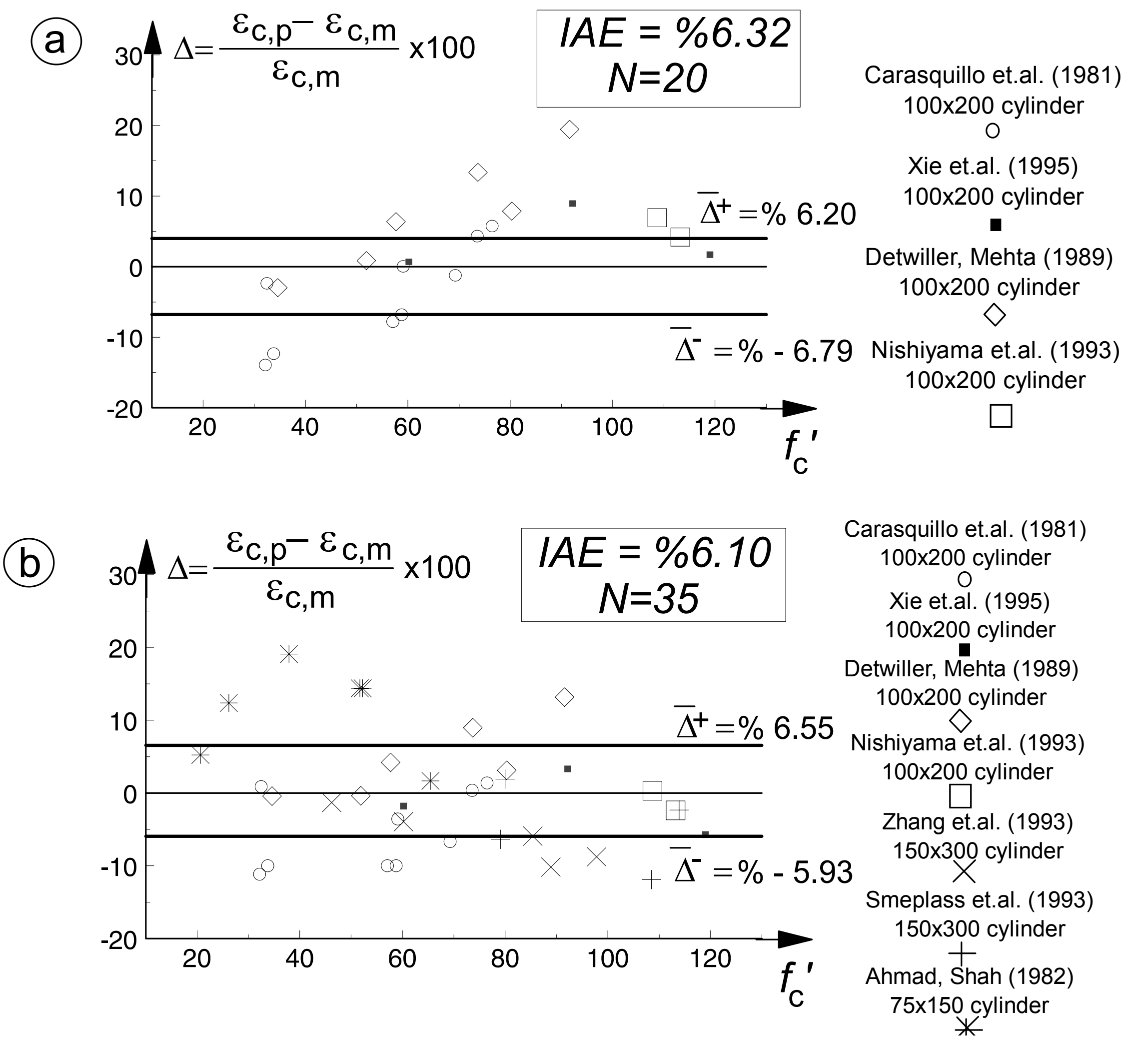


Fig.C - Error Analysis for Attard-Setunge's Equation (a) and for Arıoğlu's Equation (b)

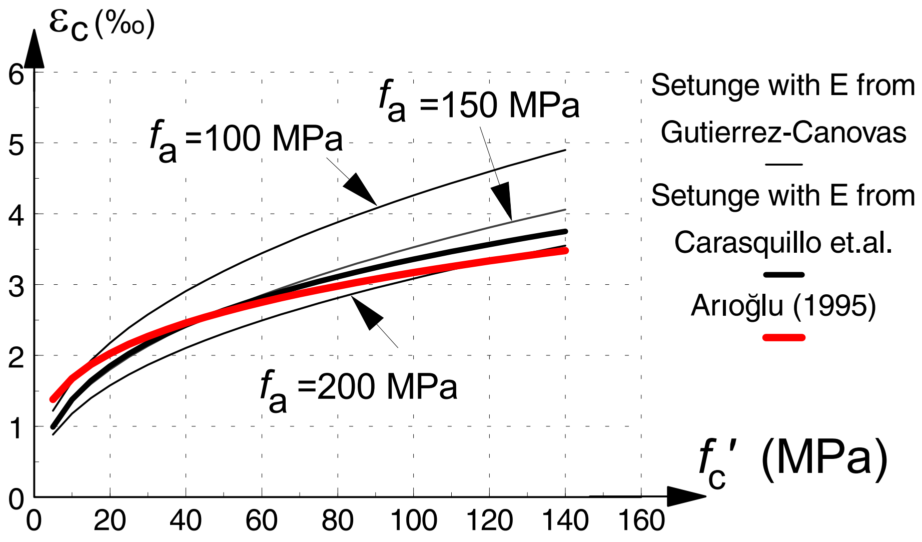


Fig.D - Strain at Peak Stress with Different "$E_c$" Equations

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