Discussion of "Strain of concrete at peak compressive stress for a wide range of compressive strengths" by B.de Nicolo, L.Pani and E.Pozzo

Ergin ARIOĞLU

Mining Engineering Department, Mining Faculty, Istanbul Technical University, Maslak, Istanbul, Turkey

I read with great interest and care this study by B. de Nicolo, L.Pani and E. Pozzo [1] in which they put forward a remarkable relationship for estimating the strain in concrete at peak compressive strength, and I acknowledge the authors for their invaluable article.

In the literature several expressions have been proposed to represent the complete stress-strain relationship of plain concrete in uniaxial compression [2-5]. From these studies it is obvious that the shape of the uniaxial stress-strain curve is influenced markedly by the following parameters:

- size (slenderness ratio: height/diameter) and shape of the specimen tested;
- characteristics of the test machine (stiffness and strain rate);
- type of strain measurement; and
- properties of the concrete (strength, water/cement ratio, cement content, mechanical properties of the coarse aggregate used and aggregate volume concentration, curing conditions and age).

To be able to define the uniaxial stress-strain curve properly, it is essential to have detailed information regarding the effects of the above-mentioned factors. To the author's knowledge, in defining the curve under consideration the strain at peak stress plays the major role because of the fact that this parameter includes the effects of several factors such as the size and shape of the specimen and other physical concrete characteristics [7].

This letter is directed towards setting up an empirical equation between the strain at peak stress and the compressive strength and volume of the specimen using multiple regression analysis. In the following analysis, experimental data employed were obtained from Table I compiled by Carreira and Chu in 1985 [4]. In addition, the proposed equation for the determination of the strain at peak stress is checked against the equation reported by Collins, Mitchell and MacGregor in 1993 [7] and by de Nicolo, Pani and Pozzo [1].

Experimental data corresponding to the strain at peak stress (compressive strength), the compressive strength and the volume of the specimen, from Table I prepared by Carreira and Chu [4] covering a broad range of normal weight concrete properties and testing conditions, were used for multiple regression analysis. According to this analysis, the strain at peak stress can be expressed in the following form

$$\boldsymbol{\varepsilon_0 = 1.753\, f_c'^{\,0.27756}\, V^{-0.09314}} \qquad \textbf{(1)}$$

with r = 0.874 and N = 41. Here $\mathbf{f_c'}$ is the peak stress or compressive strength (MPa), $\boldsymbol{\varepsilon_0}$ is the strain where $\mathbf{f_c}$ reaches $\mathbf{f_c'}$ (‰*)*, **V** is the volume of the concrete test cylinder (cm^3), r is the correlation coefficient, and N is the number of data items used.

As can be seen, there is a very significant interrelationship between the strain at peak stress $\boldsymbol{\varepsilon_0}$, and the compressive strength $\mathbf{f_c'}$ and the volume **V** of the cylindrical specimens tested. For constant compressive strength, $\boldsymbol{\varepsilon_0}$ increases considerably with decreasing V. This may be expressed by the fact that the friction between the test machine's bearing plates and the end faces of the specimen tested decreases with decreasing diameter of the specimen, giving rise to an increase in the transverse strain, i.e., ΔV increases, then the longitudinal strain to be experienced increases (ΔV is the volumetric change).

Knowing only the specified compressive strength and the volume of the cylindrical specimen, the strain at peak stress can be determined easily by making use of Equation 1. Then, the stress-strain curve for design conditions can be defined selecting one of the proposed equations in the literature.

Values of the strain at peak stress predicted by Equation 1 are compared with experimental data in Fig. 1 using a 1: 1 technique. The average deviation ($\pm\overline{\Delta} = 10\%$) estimated can be considered to be acceptable. To assess the accuracy of Equation 1 against control values, data reported by Arıoğlu [8] and Wang, Shah and Naaman

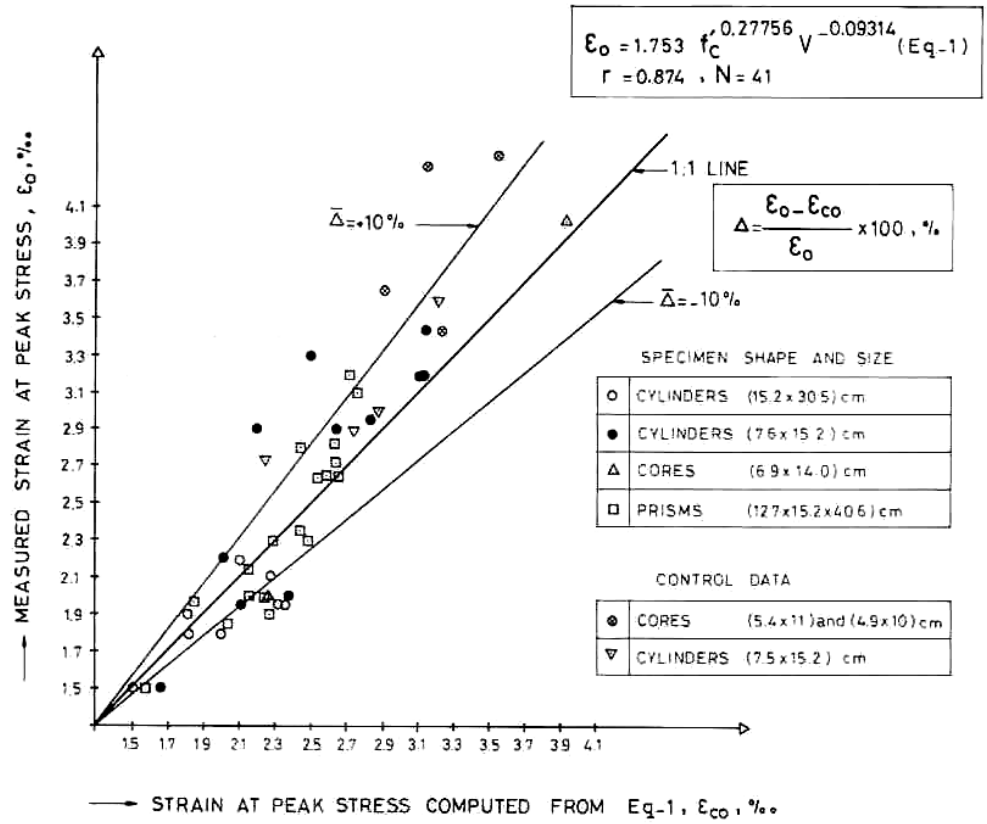

Fig. 1 Comparison between experimental values of the strain at peak stress and those calculated from Equation 1, using a 1:1 technique.

[3] which were not employed in the original analysis, were compared with the average deviation bands ($\pm\bar{\Delta}$). As is demonstrated in Fig.1, as a rule, the additional data under review appear to be within the limits of the average deviation computed for N= 41 data. It can be said that the measured values for especially small cores such as 5 cm x 10 cm are higher than those predicted by Equation 1 due to the fact that small cores contain more fissures resulting from coring procedures, giving rise to larger strains when subjected to uniaxial compression.

According to Collins, Mitchell and McGregor [7], the strain at peak ε_o can be determined from

$$\varepsilon_o = \frac{f'_c}{E_c} \cdot \frac{n}{n-1} \tag{2}$$

in which n is a curve fitting factor. Its value can be estimated from the following empirical expression.

$$n = 0.8 + \frac{f'_c \text{ (MPa)}}{17} \tag{3}$$

E_c is the elastic secant modulus (MPa). One of the empirical relationships proposed for predicting the elastic secant modulus is

$$E_c = 3320\, f_c'^{0.5} + 6900 \tag{4}$$

where f_c' and E_c are in MPa.

Putting Equations 3 and 4 into Equation 2 and numerically rearranging it, one obtains

$$\varepsilon_o = \frac{f'_c}{3320\,(f'_c)^{0.5} + 6900} \cdot \frac{0.8 + 0.0588\, f'_c}{0.0588\, f'_c - 0.2} \tag{5}$$

By making use of Equation 5, the strain at peak stress for normal weight concrete can be found. Table 1 displays the result of a comparison made between Equations 1 and 5. As can be seen, the values predicted by Equation 1 based upon a statistical analysis containing compressive strengths varying from about 8 MPa and 140 MPa.

The correlations proposed by de Nicolo, Pani and Pozzo [1] and those in this letter are compared in Fig. 2. In Equation 1 the specimen volume was taken to be V= 5534 cm^3 corresponding to a 152 cm x 30.5 cm (6 inx12 in) cylindirical specimen. As can be seen, there is acceptable agreement between relationships.

Table 1 Comparison of Equations 1 and 5.

	f_c' (MPa)						
	20	**40**	**60**	**80**	**100**	**120**	**140**
ε_o from Eq. 1 (%o)	1.86	2.10	2.39	2.65	2.93	3.18	3.41
ε_o from Eq. 5 (%o)	1.80	2.19	2.44	2.67	2.82	2.96	3.10
$\Delta = \frac{\text{Eq.1} - \text{Eq.5}}{\text{Eq.1}} \text{x100 (\%)}$	3.20	-4.29	-2.09	-0.75	3.75	6.92	9.09

In brief, there is a very significant empirical relationship between the strain at peak stress, the compressive strength and the volume of a concrete specimen (Equation 1). The results of the equation proposed in this study are found to be in very good agreement with the values (Equation 5) reported by Collins, Mitchell and MacGregor [7] (Table 1) and de Nicolo, Pani and Pozzo [1]. Equation 1 for predicting the strain at peak stress apeears also to be representative of the test data corresponding to very high strength (e.g., 140 MPa) concrete.

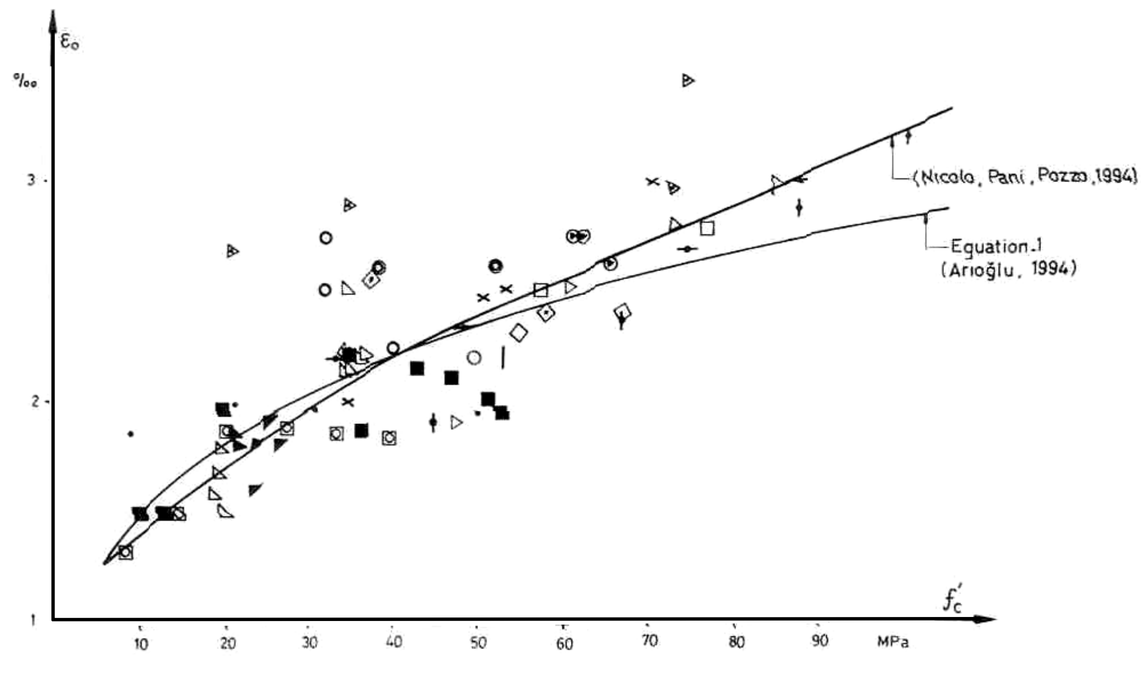

Fig. f_c' - ε_o data of de Nicolo, Pani and Pozzo (see [1], Fig.4) shown with values calculated using Equation 1.

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Reply to the discussion of Ergin Arıoğlu

E.POZZO
Universita di Cagliari, Dipartimento Ingegneria Strutturale, Cagliari, Italy

First, we wish to thank Professor Arıoğlu for his kind words in appreciation of our study.

As regards his discussion, we would like to point out that to include data on test pieces aged only a few days with data on specimens aged 28 days or more, as included by Professor Arıoğlu, could lead to some fundamental aspects of the phenomenon being lost. The reason is that the poor comparability of the data lead to excessive dispersion. It is for this reason that we have included only ageing of not less than 28 days. Coming to Arıoğlu's idea, it can be summarized as follows : for concrete specimens of equal strength , the peak strain values may seem different if they are of different volumes (a larger contact surface with the press plates is not the same thing as having a larger volume). The introduction of volume in the expression of the stress-strain curve of simple concrete may therefore represent a correction that should be able to improve its definition. To test this, we report both Arıoğlu's and our curves for a few ranges of volumes V (Fig.1) but these give no indications to confirm or refute this hypothesis, since the quantity of data available in some cases is not statistically significant.

However, we believe that Professor Arıoğlu's intuition may be valid, and that new studies should be carried out along these lines to improve the definition of the constitutive curve of concrete.

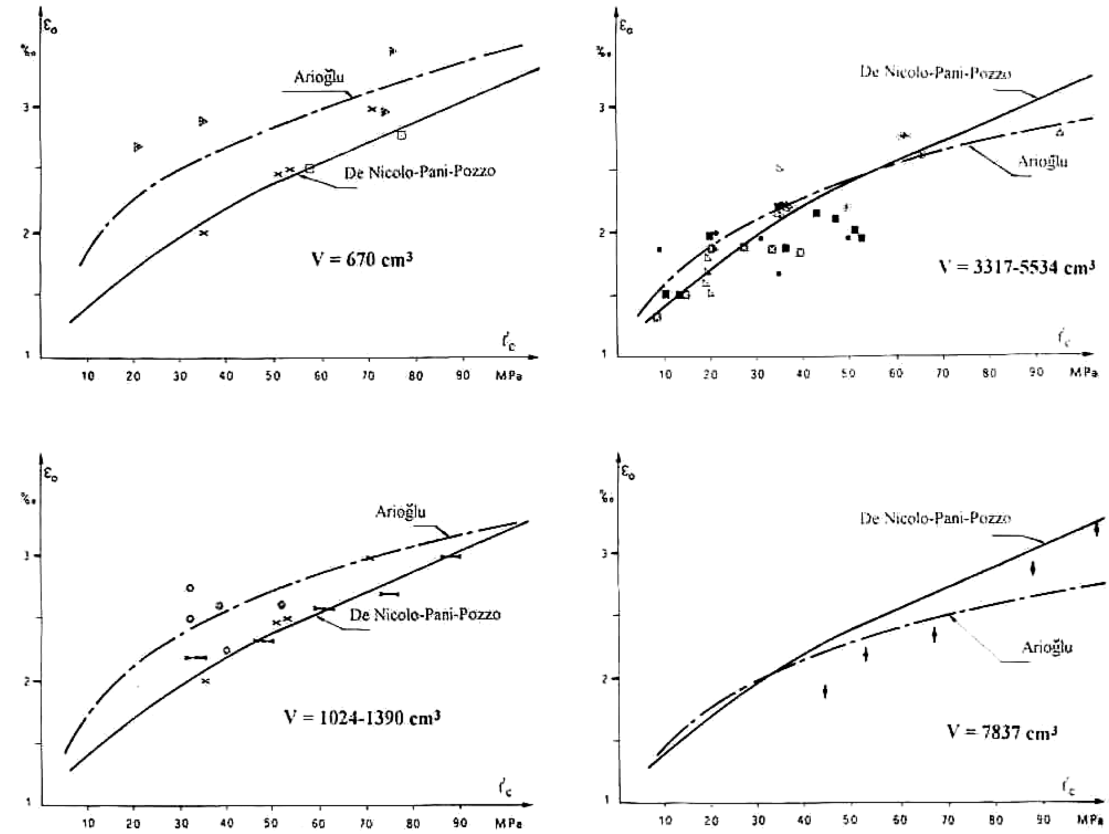

Fig. 1 Relationship proposed by Arıoğlu and by de Nicolo, Pani and Pozzo