Evaluation of Strength Criteria for Very-High-Strength Concretes under Triaxial Compression
by Z. Canan Girgin, Nihal Arıoğlu, and Ergin Arıoğlu

In this study, the empirical relationships proposed in the current literature were verified to assess their validity for compressive strengths between 60 and 132 MPa (8700 to 19,145 psi). A relationship was suggested to predict the ultimate strength of very high-strength concrete under triaxial compression. Furthermore, strength criteria essentially developed to estimate the ultimate strength of intact rock were applied to the concrete. The results of triaxial tests were employed to verify the applicability of these strength criteria for high-strength concrete. It is interesting to note that the strength criteria developed for intact rock can also be applied to the assessment of ultimate strength and failure curve in high-strength concretes. Knowing only the cylinder compressive strength and the ratio of tensile-to-compressive strength, the failure envelope can be successfully evaluated by means of these criteria without performing triaxial tests in high-strength concretes.

Keywords: compressive strength; high-strength concrete; splitting tensile strength; ultimate strength.

INTRODUCTION

High-strength concrete (HSC), having the cylinder compressive strength higher than 40 MPa (5801 psi), with enhanced properties compared with normal concrete (for example, workability, less time-dependent deformations such as shrinkage, creep under static, dynamic and fatigue loading, and durability), is increasingly used in high-rise buildings, prestressed bridge girders, and offshore oil platforms. Higher strength and less time-dependent deformation enable smaller sized structural elements. This means considerable reduction in material and labor cost as well as indirect cost-saving due to larger rentable floor space in high-rise buildings.

Compressive strengths up to 120 MPa (17,400 psi) are currently used successfully in high-rise building construction. But, in spite of the remarkable developments in high-strength and durable concretes, there is still plentiful scope for a better understanding of design parameters. For example, several empirical strength criteria on the triaxial behavior of concretes have been proposed following Richart et al.’s pioneering investigation. When examining the strength criteria more closely, it is apparent that some of these relationships were defined only at the compression region of the strength criterion and are not valid at the tensile region. While it was debated in the past that concrete is weak in tension and its tensile strength can be assumed to be zero, nowadays, a strength criterion should exist both in compressive and tensile regions to complete it, that is, high-strength concrete has considerable tensile strength and it may not be correct to neglect the tensile strength entirely.

The aim of this study is to improve the current knowledge on the ultimate confined strength of high-strength concretes subjected to triaxial compression. This study is mainly comprised of three parts. The first part re-evaluates the reliability of the significant empirical strength relationships proposed in the literature so far. Triaxial test data, which have cylinder compressive strength ranging from 60 to 132 MPa (8700 to 19,145 psi), reported by Xie et al.3 as well as Attard and Setunge4 were employed for the analysis conducted in this part. The second part of this study deals with Hoek-Brown5 and Johnston’s6 strength criteria developed for intact rocks to be extended to high-strength concretes. These criteria can be successfully applied in tension-compression region as well. Knowing only the cylinder compressive strength without carrying out any triaxial tests, it is possible to predict the failure envelope and the ultimate strength of high-strength concrete. The verification of strength criteria was also presented demonstratively and furnished through a numerical example.

RESEARCH SIGNIFICANCE

The empirical strength criteria to predict the ultimate strength of confined concrete in the current literature were verified in this study. In very-high-strength concretes with cylinder compressive strengths ranging from 60 to 132 MPa (8700 to 19,145 psi) a very close agreement was not obtained between the experimental data and the predicted results through some expressions. An empirical relationship was suggested to assess the ultimate strength $f_1$ of concrete in this strength range.

This study also finds out that Hoek-Brown’s and Johnston’s strength criteria developed for intact rocks can be applied to the assessment of ultimate strengths in very-high-strength concretes as well. Knowing only the cylinder compressive strength, the failure envelope of confined concrete can be successfully estimated without performing any triaxial compression test. This is especially important if any comprehensive information on reliability of an expression is not available. Moreover, this knowledge could reduce or eliminate costs of triaxial testing programs in very-high-strength concrete.

RE-EVALUATION OF EMPIRICAL STRENGTH CRITERIA FOR HIGH-STRENGTH CONCRETE

The current empirical strength criteria proposed for confined concrete were generally derived from the results of column tests under axial compression. In these tests, the confinement, that is passive in nature, is provided by the transverse reinforcement of the column. The ratio of confining pressure to cylinder compressive strength $f_c/f_c$ is practically less than approximately 0.20. Confining pressure
The results of experimental data were used to re-evaluate the reliability of the significant empirical strength criteria (Table 1) proposed for high-strength concretes in the current literature. The reliability of the results can be verified with the integral absolute error (IAE) criterion (Eq. (1)), which is statistically very sensitive and measures the relative deviations of data from the regression equation.

Xie et al. and Attard and Setunge conducted a series of triaxial tests on φ100 x 200 mm (φ4 x 8 in.) cylinder samples. Xie et al. investigated the high-strength concrete with cylinder compressive strength ranging from 60 to 119 MPa (8700 to 17,260 psi) under confining pressures ranging between 0 to 50 MPa (0 to 7250 psi). Attard and Setunge carried out the experiments for high-strength concretes within the range of 60 to 132 MPa (8700 to 19,145 psi) having water-cementitious material ratios (w/cm) of 0.26, 0.30, 0.35, and 0.45. The maximum confining pressure \( f_r \) in their test program was approximately 20 MPa (2900 psi) for \( f_r/f_c \) ratio of 0.15.

The corresponding ratio can be regarded as intermediate confining pressure of 5 to 50 MPa (720 to 7250 psi). Referring to standard triaxial tests, the confinement provided by fluid pressure is active. Richart et al. evaluated the responses under passive confinement and noticed the resembling behavior.

### Table 1—Summary of relationships proposed for ultimate compressive strength and calculated integrated absolute error (IAE)

<table>
<thead>
<tr>
<th>Authors</th>
<th>Equations</th>
<th>Range</th>
<th>Confinement type and remarks</th>
<th>In range of all data IAE, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Richart et al.</td>
<td>( \frac{f_u}{f_c} = 1 + 4.1\frac{f_r}{f_c} )</td>
<td>20 MPa ( \leq f_c \leq 50 ) MPa</td>
<td>Active</td>
<td>10.7</td>
</tr>
<tr>
<td>Balmer</td>
<td>( \frac{f_u}{f_c} = (1 + 9.175\frac{f_r}{f_c})^{0.33} )</td>
<td>20 MPa ( \leq f_c \leq 50 ) MPa</td>
<td>Passive</td>
<td>6.1</td>
</tr>
<tr>
<td>Martinez et al.</td>
<td>( \frac{f_u}{f_c} = 1 + 4.2\frac{f_r}{f_c} )</td>
<td>21 MPa ( \leq f_c \leq 69 ) MPa</td>
<td>Passive, confined by spiral reinforcement without concrete cover and longitudinal reinforcement</td>
<td>11.5</td>
</tr>
<tr>
<td>Fafitis and Shah</td>
<td>( f_u = f_{co} + K_1 \cdot f_c \cdot f_r \cdot f_{cr} \cdot f_{co} = 6.7 (K_2 \cdot f_c \cdot f_r)^{0.17} ) For ( 1 &lt; f_r &lt; 8 ) MPa; ( 60 \leq f_c \leq 124 ) MPa; Comment: ( f_c = f_{co} + 6.7f_c^{0.83} )</td>
<td>26 MPa ( \leq f_c \leq 66 ) MPa</td>
<td>Passive confinement generated by transverse reinforcement</td>
<td>25.1</td>
</tr>
<tr>
<td>Saatcioglu and Razvi</td>
<td>( f_u = f_c + K_1 \cdot f_c \cdot f_r \cdot f_{cr} \cdot f_{co} = 6.7 (K_2 \cdot f_c \cdot f_r)^{0.17} ) For ( 1 &lt; f_r &lt; 8 ) MPa; ( 60 \leq f_c \leq 124 ) MPa; Comment: ( f_c = f_{co} + 6.7f_c^{0.83} )</td>
<td>In case of closely spaced spirals in circular columns ( K_2 \approx 1 )</td>
<td>10.8</td>
<td></td>
</tr>
<tr>
<td>Setunge et al.</td>
<td>( f_u = 9.05f_c ) For ( 1 &lt; f_r &lt; 8 ) MPa; ( 60 \leq f_c \leq 124 ) MPa; Comment: ( f_c = f_{co} + 6.7f_c^{0.83} )</td>
<td>90 MPa ( \leq f_c \leq 132 ) MPa</td>
<td>Active, triaxial tests are on high-strength concrete samples. Relationships correspond to silica fume, no silica fume, and normal strength concrete, respectively.</td>
<td>6.0</td>
</tr>
<tr>
<td>Xie et al.</td>
<td>( f_u = 1 + 2.4\left(\frac{f_r}{f_c}\right)^{0.7} )</td>
<td>30 MPa ( \leq f_c \leq 120 ) MPa</td>
<td>Passive, test results are on circular and square large-scale columns</td>
<td>12.1</td>
</tr>
<tr>
<td>Bing et al.</td>
<td>( f_u = f_{co} - 2\sqrt{1 + 7.94\frac{f_c}{f_{co}} - 2\frac{f_c}{f_{co}} - 2\frac{f_r}{f_{co}} - \frac{f_c}{f_{co}} - 2\frac{f_c}{f_{co}} - \frac{f_r}{f_{co}}} ) If ( f_{co} \leq 52 ) MPa, ( \alpha_1 = (21.2 - 0.35f_{co})\frac{f_r}{f_{co}} ) If ( f_{co} &gt; 52 ) MPa, ( \alpha_1 = 3.1\frac{f_r}{f_{co}} ) Modification of Mander et al. Model is based on theoretical ultimate strength surface, taking tensile and compressive meridians into consideration.</td>
<td>30 MPa ( \leq f_c \leq 120 ) MPa</td>
<td>For ( f_{co} = 0.85f_c ) 23.8</td>
<td></td>
</tr>
<tr>
<td>Current study</td>
<td>( f_u = 1 + 4.08\left(\frac{f_r}{f_c}\right)^{0.85} )</td>
<td>60 MPa ( \leq f_c \leq 132 ) MPa</td>
<td>For ( f_{co} = f_c ) and ( f_c = 17.7 )</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Note: \( f_{co} \) equals in-place unconfined compressive strength of concrete (ratio of unconfined strength in-place \( f_{co} \) in column to standard cylinder strength \( f_{co} \) is generally taken as 0.85 for normal strength. Tests performed on concrete specimens of same size and shape show range of variation between 0.85 and 1.0 by Razvi and Saatcioglu. For high-strength concrete, ratio may be assumed equal to one without causing considerable error, that is, \( f_{co} = f_c \)). \( \alpha_1 \) equals modification factor; \( f_{co} \) equals splitting tensile strength of 6 x 12 in. cylinder, MPa; \( r \) equals correlation coefficient of regression equation; and \( n \) equals number of data test points.
where \( O_i \) is the observed value and \( P_i \) is the predicted value from the regression equation. A range of the IAE from 0 to 10% may be regarded as the limits for an acceptable regression equation.

**Results of re-evaluation and discussion**

The findings of the re-evaluation based on IAE values are given in Table 1. Some of the empirical relationships are plotted in Fig. 1 in a normalized form (\( f/c \)). The following observations can be made from Table 1 and Fig. 1.

- The IAE is at a maximum (25.1%) for Fafitis and Shah’s equation, 12 12.1% for Légeron and Paultre’s equation, 16 10.8% for Saatcioglu and Razvi’s equation, 13,14 10.7% for Richart et al.’s classical equation, 2 6.0 to 10.2% for Setunge et al.’s equation, 15 6.1% for Balmer’s equation, 10 minimum (4.8%) for Xie et al.’s equation 3 and for the proposed equation in this study.
- The IAE for the equation of Bing et al.’s 17 seems to be affected from the ratio of unconfined strength in-place to cylinder compressive strength \( f_c/f_c \). When this ratio is taken as 0.85, adapted by ACI Committee 363, 20 the IAE value is estimated as 23.8%.
- The type of confinement is believed to have no influence on the IAE level.
- The IAE seems to be largely related to the form of the relationship chosen. For example, the proposed equation in this study with a minimum IAE of 4.8% conforms well to test results. Whereas the linear relationships, which are usually employed in the codes, do not agree accurately with the test data on high-strength concrete.
- The proposed equations in this study conform well to the concretes made with three types of coarse aggregate having different compressive strengths and mineralogical properties. In other words, the type of aggregate has no significant effect on the normalized strengths (\( f/c \) versus \( f/c \)). A similar observation was made by Setunge et al. 15 as well.

**PRINCIPLES OF HOEK-BROWN AND JOHNSTON STRENGTH CRITERIA**

**Hoek-Brown criterion**

Hoek and Brown 2 proposed an empirical strength criterion for rocks \((f_c \geq 20 \text{ MPa})\). This criterion can be written in the following form.

\[
  f_1 = f_r + (s \cdot f_c + m \cdot f_c \cdot f_t)^{1/2}
\]  

(2a)

where \( f_1 \) is the major principal stress at failure (ultimate strength), \( f_r \) is the minor principal stress (confining pressure), and \( f_c \) is the uniaxial compressive strength of intact rock specimens. Herein, \( m \) is a material constant governing the curvature of \( f_1 \) versus \( f_c \) curve and its value depends on the type of rock, for example, the representative value \((s)\) of \( m \) for sandstone and quartzite is 15. The other material constant \( s \) describes the discontinuities (for example, rock joints) and ranges from 0 for heavily jointed rocks to 1 for intact rocks. Providing that the volume of concrete contains no discontinuity in macroscale, the concrete can be treated as intact rock material.

Eq. (2a) may be expressed as

\[
  IAE = \sum \left[ \frac{(O_i - P_i)^2}{\Sigma O_i} \right]^{1/2} \times 100
\]  

(1)

**Johnston criterion**

Johnston 6 proposed a strength criterion (Eq. 6a) for all intact geomechanical materials, ranging from 0.008 MPa (1.1 psi) (lightly overconsolidate clays) to 600 MPa (87,000 psi) (extremely hard rocks)
As discussed by Johnston\textsuperscript{6} and supported experimentally, the $M/B$ ratio in Eq. (7a) and (7b) seems to be a function of both the uniaxial compressive strength $f_c$ and the type of rock.

**Application of strength criteria to high-strength concrete**

This section will emphasize the Hoek-Brown and Johnston strength criteria on how to extend to concrete (for relatively hard intact rocks, it was reported\textsuperscript{22} that two criteria are in a good agreement).

**Application of Hoek-Brown strength criterion**—The procedure steps for concrete are summarized in the following.

- For the concrete with no macro-discontinuities, the material constant $s$ is taken as $s = 1$ (that is, intact rock).
- Employing a series of triaxial test data, for the uniaxial (cylinder) compressive strength level $f_c$, the material constant $m$ is calculated by means of the linear regression analysis (Eq. (3)) between $Y$ and $X$. Thus, the failure envelope (Eq. (2a)) between the pairs of $f_1$ and $f_c$ values can be constructed easily.
- If the triaxial test data are not available anyway, or if eliminating the cost of triaxial testing is preferred, a reliable and practical approach can be adopted. The failure envelope is set up by using $m$ assigned through aforementioned regression analysis.

**Application of Johnston strength criterion**—The procedure is outlined step-by-step in the following.

- For the test data pairs $f_1/f_c$ and $f_1/f_c$ from a series of triaxial tests on intact samples of concrete, $B$ can be statistically estimated through Eq. (9) derived from Eq. (6b)

\[
B = \frac{\log \frac{f_1}{f_c}}{\log \left(1 + \frac{f_c}{0.9f_{1sp}}\right)}
\]  

- The failure envelope (Eq. (6b)) between the pairs of $f_1$ and $f_c$ values is defined.

In the absence of triaxial test data, by knowing only compressive strength and tensile strength, the procedure to set up the failure envelope consists of the following steps.

- $B$ is assigned from Eq. (8).
- Employing $f_1/f_{1sp}$ ratio in Eq. (6b), the ultimate strengths $f_1$ standing for the definite confining stress levels $f_c$ can be estimated and the failure envelope is constructed.

If there is no knowledge of the tensile strength ($f_t$, $f_{1sp}$) as well, by knowing only compressive strength, an alternative solution through Eq. (10)\textsuperscript{9} can be applied

\[
\frac{f_{1sp}}{f_c} = 0.387f_c^{-0.370}, f_c \text{ in MPa}
\]  

\[
(n = 326 \text{ data}, r = 0.980, 4 \text{ MPa} < f_c < 130 \text{ MPa})
\]

\[
(580 < f_c < 18,855 \text{ psi})
\]

where $n$ is number of data and $r$ is the correlation coefficient. If it is required the conversion\textsuperscript{21} $f_t \approx 0.9f_{1sp}$ can be made as well. The rest of procedure is similar to the previous one.
### Table 2—Prediction of material constant $m$ according to Hoek-Brown criterion for ultimate compressive strength of high-strength concrete

<table>
<thead>
<tr>
<th>Compressive strength in set $f_c$, MPa (psi)</th>
<th>Predicted $m$ from Eq. (3)</th>
<th>$r$</th>
<th>Data source</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.2 (8730)</td>
<td>12.761</td>
<td>11</td>
<td>0.998</td>
<td>Xie et al.$^3$</td>
</tr>
<tr>
<td>92.2 (13,370)</td>
<td>12.759</td>
<td>11</td>
<td>0.996</td>
<td></td>
</tr>
<tr>
<td>119 (17,260)</td>
<td>10.910</td>
<td>11</td>
<td>0.991</td>
<td>Attard and Setunge$^4$</td>
</tr>
<tr>
<td>60 (8700)</td>
<td>14.682</td>
<td>4</td>
<td>0.999</td>
<td>Hornfel</td>
</tr>
<tr>
<td>96 (13,925)</td>
<td>7.467</td>
<td>3</td>
<td>0.971</td>
<td></td>
</tr>
<tr>
<td>100 (14,505)</td>
<td>8.828</td>
<td>7</td>
<td>0.990</td>
<td></td>
</tr>
<tr>
<td>110 (15,955)</td>
<td>7.864</td>
<td>6</td>
<td>0.970</td>
<td></td>
</tr>
<tr>
<td>118 (17,115)</td>
<td>10.504</td>
<td>3</td>
<td>0.993</td>
<td></td>
</tr>
<tr>
<td>120 (17,405)</td>
<td>12.989</td>
<td>9</td>
<td>0.993</td>
<td></td>
</tr>
<tr>
<td>126 (18,275)</td>
<td>10.926</td>
<td>3</td>
<td>0.999</td>
<td>Basalt</td>
</tr>
<tr>
<td>132 (19,145)</td>
<td>9.261</td>
<td>3</td>
<td>0.999</td>
<td>Basalt</td>
</tr>
</tbody>
</table>

Average values of $m$ for all 71 test data
- Arithmetic average: 10.81
- Weighted average: 11.265

$m$ corresponding to minimum IAE
- $m = 13.0$, $\bar{m} = 4.82, \Delta = -4.07, -6.74$

Notes: $f_c$ equals ultimate compressive strength, $f_p$ equals uniaxial (cylinder) compressive strength, $F_{1,2}$ equals material constant; $\bar{m}$ equals average deviation; $\Delta$ equals deviation; $|\Delta| = (f_{1,2} - f_c)^2/\sigma_{f_c}^2 (100, \%)$; $f_{1,2} = f_c$, $f_{1,2}$ equals, respectively, observed and predicted ultimate compressive strength; $r$ equals correlation coefficient; and $n$ equals number of data in set.

### Remarks on criteria

The application to concrete of Hoek-Brown and Johnston’s strength criteria has been verified by using Xie et al.$^3$ and Attard and Setunge’s$^4$ 71 triaxial test data on high-strength concrete. The cylinder compressive strength $f_c$ level of test data is within the range 60 to 132 MPa (8700 to 19,145 psi). The values $m$ and $B$ for each compressive strength level of triaxial data were computed individually and tabulated in Tables 2 and 3. The $m$ constants of the Hoek-Brown criterion were predicted from Eq. (3) and the $B$ coefficients of Johnston criterion were predicted from Eq. (8) and (9) as well. In addition, proposing one $m$ and $B$ value covering the whole range (60 to 132 MPa) was adopted to make reliable and practical estimations. Arithmetic and weighted average values of $m$ and $B$ were also computed; furthermore, the constant values of $m$ and $B$ were determined in such a way as to give the smallest IAE value.

Some findings from Tables 2 and 3 are summarized as follows.

- For the range 60 to 132 MPa (8700 to 19,145 psi), the material constant $m$ was predicted as 13 with a minimum IAE value of 4.8%. (This $m$ value for intact rocks falls into the rock type$^2$: sedimentary rock type class: clastic, sandstone-siltstone; texture: medium-fine. When the non-clastic class is considered, the same value corresponds to the carbonate rocks such as the brecia-sparitic limestone.)
- The $B$ value defining the confinement effectiveness in the Johnston strength criterion was estimated as 0.5 with a minimum IAE value of 5.8%. There is a good agreement between the $B$ values computed from triaxial test data (Eq. (9)) and directly estimated from Eq. (8) by knowing only $f_c$. When $B = 0.5$, the Johnston criterion becomes similar to the Hoek-Brown criterion.
- Both criteria seem to be in a good agreement with the triaxial data under consideration.

These findings suggest that the failure envelope for high-strength concrete can be established with a reasonable accuracy by knowing only the cylinder compressive strength as well.

### Table 3—Prediction of material constant $B$ according to Johnston criterion for ultimate compressive strength of high-strength concrete

<table>
<thead>
<tr>
<th>Compressive strength in set $f_c$, MPa (psi)</th>
<th>$B$ from Eq. (8)</th>
<th>$B$ from Eq. (9) (average)</th>
<th>No. of data in set, $n$</th>
<th>Data source</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>60.2 (8730)</td>
<td>0.607</td>
<td>0.588</td>
<td>11</td>
<td>Xie et al.$^3$</td>
<td></td>
</tr>
<tr>
<td>92.2 (13,370)</td>
<td>0.576</td>
<td>0.562</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>119 (17,260)</td>
<td>0.557</td>
<td>0.500</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 (8700)</td>
<td>0.607</td>
<td>0.614</td>
<td>4</td>
<td>Attard, Setunge$^4$</td>
<td></td>
</tr>
<tr>
<td>96 (13,925)</td>
<td>0.573</td>
<td>0.404</td>
<td>3</td>
<td>Hornfel</td>
<td></td>
</tr>
<tr>
<td>100 (14,505)</td>
<td>0.570</td>
<td>0.405</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>110 (15,955)</td>
<td>0.563</td>
<td>0.507</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>118 (17,115)</td>
<td>0.558</td>
<td>0.468</td>
<td>3</td>
<td>Basalt</td>
<td></td>
</tr>
<tr>
<td>120 (17,405)</td>
<td>0.556</td>
<td>0.548</td>
<td>9</td>
<td>Basalt</td>
<td></td>
</tr>
<tr>
<td>126 (18,275)</td>
<td>0.553</td>
<td>0.470</td>
<td>3</td>
<td>Basalt</td>
<td></td>
</tr>
<tr>
<td>132 (19,145)</td>
<td>0.549</td>
<td>0.530</td>
<td>3</td>
<td>Basalt</td>
<td></td>
</tr>
</tbody>
</table>

Average values of $B$ for all 71 test data
- Arithmetic average: 0.570
- Weighted average: 0.572

$B$ value making minimum IAE
- $B = 0.500$, $\bar{B} = 0.482, \Delta = +3.4, –10.0$

Note: $B$ equals material constant, for other notations, see Table 2.

### Numerical example

The application of empirical strength criteria will be exemplified for Attard and Setunge’s$^4$ triaxial data having a cylinder compressive strength $f_c = 132$ MPa (19,145 psi) and confining stress $f_p = 15$ MPa (2175 psi).

#### Solution with proposed strength criteria—

- Application of relationship developed in this study (Table 1)

$$\frac{f_1}{f_c} = 1 + 4.08\left(\frac{f_p}{f_c}\right)^{0.83} \quad (60 \text{ MPa} \leq f_c \leq 132 \text{ MPa})$$
for \( f_r = 15 \text{ MPa} \) the ultimate strength is predicted as \( f_1 = 220.6 \text{ MPa} \) (31,995 psi).

**Application of Hoek-Brown Strength Criterion:**
Knowing the \( m \) value as 13 (Table 2) for high-strength concretes and assuming \( s = 1 \) for concrete (that is, intact rock), the failure envelope (Eq. (2a)) can be set up directly

\[
f_1 = f_r + (s \cdot f_c + m \cdot f_c \cdot f_r)^{1/2} = f_r + (132^2 + 13 \times 132f_r)^{1/2}
\]

and for \( f_r = 15 \text{ MPa} \) (2175 psi), the ultimate strength is estimated as \( f_1 = 222.7 \text{ MPa} \) (32,300 psi).

**Application of Johnston’s strength criterion:**
To predict the compressive strength to tensile strength ratio corresponding to \( M/B \) ratio in Eq. (6a), Eq. (10) is employed

\[
\frac{f_{tsp}}{f_c} = 0.387 \cdot f_c^{0.370} = 0.387 \cdot 132^{0.370} = 0.063547
\]

Then taking the inverse of \( f_{tsp}/f_c \) ratio and knowing the value of \( B \) as 0.5 from Table 3, the failure envelope can be written

\[
f_1 = f_r + (s \cdot f_c + m \cdot f_c \cdot f_r)^{1/2} = f_r + (132^2 + 13 \times 132f_r)^{1/2}
\]

where \( f_{tsp}/f_r \) ratio signifies \( M/B \) ratio. For \( f_r = 15 \text{ MPa} \) (2175 psi) the ultimate strength \( f_1 \) is calculated as 228.1 MPa (33,080 psi).

**CONCLUSIONS**

The following conclusions can be drawn from this study:

- Some of the empirical strength relationships proposed in the concrete literature (Table 1) disagree with the triaxial test data on high-strength concretes (Fig. 1). The widely adapted Bing et al. equation, which is derived on the basis of the failure surface for high-strength concrete in the compression-compression region, may have considerable errors. This result...
implies that the tensile strength plays a significant role in the nonlinear characteristics of the failure envelope for high/very-high-strength concrete.

- An empirical relationship (Table 1) is presented to predict the failure envelope of the high-strength concrete having cylinder compressive strength within the range 60 to 132 MPa (8700 to 19,145 psi). This model includes a power index indicating nonlinearity. When the value of power index becomes closer to the unit, the relationship should become more linear. In this case, the relationship is really identical to the widely accepted Richart et al. equation:

\[ f_c = f_y \left( \frac{f_t}{f_y} \right)^n \]

For intact rocks, the empirical strength criteria introduced by Hoek and Brown\(^2\) and by Johnston\(^6\) were applied to confined concrete and verified to describe the failure envelopes of high-strength concretes (Tables 2 and 3). These criteria essentially are valid not only in the compressive region and but also in the tensile region as well (Fig. 2). For the range under consideration, the material constant \( m \) of the Hoek-Brown criterion was predicted as 13.

- **Hoek-Brown and Johnston strength criteria** provide the opportunity to analytically estimate the failure envelope of confined concrete by knowing only \( f_y \) without performing any triaxial compression test. This is especially important if any comprehensive information on the reliability of an expression is not available anyway. Moreover, this knowledge could eliminate the costs of triaxial testing programs and offer a practical opportunity to reliably predict the ultimate strength \( f_u \). Applicability to high-strength concrete of three relationships under consideration in this study was exemplified for \( f_c = \) 120 MPa (17,400 psi) and 60 MPa (8700 psi) in Fig. 3 and 4 and for 132 MPa (19,145 psi)\(^8\) in the numerical example. The results agree well with the experimental triaxial data.\(^3,4\) Owing to the limited nature of the available test data, further research is required to generalize the aforementioned conclusion for concretes ranging from normal to ultra-high strength (20 to 180 MPa [2900 to 26,100 psi]).

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